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An Improved Weighted Evidence Combination Based on Tangent Similarity and Its Application in Decision-Making

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1. Introduction

Making accurate decisions in real-life scenarios is still challenging, as information collected from different sources may be imprecise and ambiguous due to the influence of multiple factors [\[27\]](#page-11-0). In order to better manage this imprecise information, many theories have been developed, such as fuzzy set and its extension theories [\[1,](#page-9-0) [4,](#page-9-1) [32\]](#page-11-1), Dempster-Shafer theory [\[20,](#page-10-0) [28,](#page-11-2) [33\]](#page-11-3), possibility theory [\[9,](#page-10-1) $\overline{40}$, neutrosophic set theory $\overline{8, 25, 34}$ $\overline{8, 25, 34}$ $\overline{8, 25, 34}$, etc. These theories have proven effective in various domains such as medical diagnosis $\left[2, 19, 29, 44\right]$ $\left[2, 19, 29, 44\right]$, clustering $\left[13, 16, 25, 26\right]$ $\left[13, 16, 25, 26\right]$, classification $\left[3, 21, 22\right]$ $\left[3, 21, 22\right]$ $\left[3, 21, 22\right]$ $\left[3, 21, 22\right]$ $\left[3, 21, 22\right]$, fault diagnosis $[14, 24]$ $[14, 24]$ $[14, 24]$, information fusion $[17, 28, 42]$ $[17, 28, 42]$ $[17, 28, 42]$ $[17, 28, 42]$ $[17, 28, 42]$, and decision-making $[18, 23]$ $[18, 23]$ $[18, 23]$.

Among them, Dempster-Shafer theory (DST), initially introduced by Dempster and expanded by

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Shafer, excels in capturing and expressing uncertainty and imprecision [\[5,](#page-10-9) [38\]](#page-12-3). It facilitates the merging of evidence even without prior information $[27]$. Furthermore, DST supports the aggregation of evidence through Dempster's rule, which adheres to associative and commutative properties, making it beneficial for fusing data from multiple sources. However, when using traditional Dempster's rule for evidence fusion, the results often lead to conclusions that violate common sense in some cases. This is mainly because the traditional Dempster fusion rules are only applicable to fusion of evidence with small evidence conflicts. Once fusion of evidence with large conflict of evidence, conclusion paradox will occur.

To address this shortcoming, a variety of solutions have been introduced by researchers [\[11,](#page-10-10) [15,](#page-10-11) [35,](#page-11-12) [41,](#page-12-4) [43\]](#page-12-5). These approaches primarily concentrate on modifying Dempster's rule or preprocessing the evidence before applying Dempster's rule. There are some alternative combination rules, such as Yager's rule [\[37\]](#page-12-6) and Dubois and Prade's rule [\[7\]](#page-10-12). Moreover, a notable method is Murphy's simple average method of evidences [\[31\]](#page-11-13). Building on Murphy's concept, Deng et al. [\[6\]](#page-10-13) proposed a Jousselme distance-based weighted average method. Lin et al. [\[14\]](#page-10-6) designed the evidential Euclidean distance to evaluate the difference between evidences. Interestingly, there are also other ways to deal with the problem, such as Xiao's belief divergence [\[36\]](#page-12-7), Liu's belief Sørensen coefficient [\[17\]](#page-10-7), Huang et al.'s belief logarithmic similarity [\[10\]](#page-10-14), Liu's evidential sine similarities [\[18\]](#page-10-8) and Lyu and Liu's Sharma-Mittal diver-gence [\[28\]](#page-11-2). Despite these efforts, the problem remains partially unsolved. Some solutions manage to alleviate the issue to an extent but inadvertently compromise essential properties like commutativity and associativity. Consequently, there remains scope for improvement in achieving a more precisely defined result in evidence fusion.

Lately, some research has focused on the tangent similarity measure within various theoretical domains such as fuzzy set theory [\[30\]](#page-11-14) and neutrosophic set theory [\[12,](#page-10-15) [39\]](#page-12-8), primarily for modeling uncertainty in information. Given its effectiveness, it is essential to integrate the tangent similarity measure into evidence theory, which could potentially address diverse challenges by enhancing the framework's ability to manage uncertainty. Therefore, this paper introduces a novel tangent similarity measure within DST. This new tangent similarity measure aims to deliver a more precise evaluation of evidence conflicts. Additionally, the reliability and practical utility of the proposed measure are substantiated through detailed mathematical proofs, demonstrating its desirable characteristics. Furthermore, an improved weighted evidence combination strategy, grounded in the proposed tangent similarity measure, is developed. The effectiveness of the decision-making method is validated through a specific application.

The structure of this paper is laid out as follows. Section 2 provides a concise overview of Dempster-Shafer theory. Section 3 introduces a novel tangent similarity measure. In Section 4, an improved weighted evidence combination method based on the new measure is detailed. The performance of the method is evaluated through an application in Section 5. The paper concludes with Section 6, summarizing the key findings.

2. Background

Dempster-Shafer theory (DST) is renowned for its robust approach to uncertainty management, surpassing traditional probability models by allowing more direct expression of informational ambiguity.

In DST, let us define $\mathbb O$ as a set containing N distinct, mutually exclusive elements, labeled as the frame of discernment (FOD):

$$
\mathbb{O} = \{O_1, O_2, ..., O_N\}
$$
 (1)

The power set of $\mathbb O,$ denoted as $2^{\mathbb O},$ includes all possible subsets of $\mathbb O:$

$$
2^{\mathbb{O}} = \{\emptyset, \{\mathcal{O}_1\}, \{\mathcal{O}_2\}, ..., \{\mathcal{O}_N\}, \{\mathcal{O}_1, \mathcal{O}_2\}, ..., \mathbb{O}\}\
$$
 (2)

DST uses a mass function, also known as a basic probability assignment (BPA), which maps each element of $2^\mathbb{O}$ to a value between [0, 1]. This function adheres to the following criteria:

- The BPA m assigns to each subset a value such that $m: 2^{\mathbb{O}} \to 1$.
- The sum of belief values for all subsets is 1: \sum $\bar{\mathcal{O}_i}$ ⊆o $m(\mathcal{O}_i)=1.$
- The belief assigned to the empty set is zero: $m(\emptyset) = 0$.

This mapping ensures that each subset \mathcal{O}_i , if it holds any positive belief value $m(\mathcal{O}_i)~>~0$, is considered a focal element.

When dealing with two independent BPAs m_1 and m_2 , Dempster's rule of combination offers a systematic approach to integrate these beliefs, defined as:

$$
m(\mathcal{O}_i) = \begin{cases} 0, & \mathcal{O}_i = \emptyset \\ \frac{\sum\limits_{\mathcal{O}_j \cap \mathcal{O}_k = \mathcal{O}_i} m_1(\mathcal{O}_j) m_2(\mathcal{O}_k)}{1 - K}, & \mathcal{O}_i \neq \emptyset \end{cases}
$$
 (3)

with

$$
K = \sum_{\mathcal{O}_j \cap \mathcal{O}_k = \emptyset} m_1(\mathcal{O}_j) m_2(\mathcal{O}_k)
$$
 (4)

where K is the conflict coefficient between m_1 and m_2 .

3. Proposed tangent similarity measure

In DST, measuring the similarities between evidences effectively is still a challenging issue. This section introduces a novel tangent similarity measure to address this challenge. Additionally, we delve into several properties of the newly proposed similarity measure to highlight its potential advantages.

Definition 1 *(Tangent similarity measure) Let* m_1 *and* m_2 *are two BBAs on* \mathbb{O} *, a new tangent similarity measure (* $GCSM$ *) between* m_1 *and* m_2 *is defined as:*

$$
TSM(m_1, m_2) = 1 - \tan\left(\frac{\pi}{8} \sum_{\mathcal{O}_i \subseteq \mathcal{O}} |m_1(\mathcal{O}_i) - m_2(\mathcal{O}_i)|\right)
$$
 (5)

Different from the previous tangent similarity measures, we use BPAs to replace the probability distribution function, thereby ensuring that the proposed tangent similarity measure can work under the framework of DST.

Theorem 1 *The proposed* T SM *satisfies the following properties:*

- *1. Symmetry:* $TSM(m_1, m_2) = TSM(m_2, m_1)$ *.*
- *2. Bounded:* $0 < TSM(m_1, m_2) < 1$.
- 3. Non-degeneracy: $TSM(m_1, m_2) = 1$ *iff* $m_1 = m_2$.

Proof 1 *For two BBAs* m_1 *and* m_2 *on* \mathbb{O} *, we have:*

$$
TSM(m_1, m_2) = 1 - \tan\left(\frac{\pi}{8} \sum_{\mathcal{O}_i \subseteq \mathcal{O}} |m_1(\mathcal{O}_i) - m_2(\mathcal{O}_i)|\right)
$$

Clearly, we can get the following:

$$
0 \leq \sum_{\mathcal{O}_i \subseteq \mathbb{O}} |m_1(\mathcal{O}_i) - m_2(\mathcal{O}_i)| \leq 2
$$

and

$$
0 \leq \frac{\pi}{8} \sum_{\mathcal{O}_i \subseteq \mathcal{O}} |m_1(\mathcal{O}_i) - m_2(\mathcal{O}_i)| \leq \frac{\pi}{4}
$$

For $tan(x)$ *,* $x \in [0, \frac{\pi}{4}]$ $\frac{\pi}{4}$], its range is $[0,1]$. Therefore, $0 \leq TSM(m_1,m_2) \leq 1$.

Proof 2 *For two arbitrary BPAs* m_1 *and* m_2 *in* \mathbb{O} *, we have:*

$$
TSM(m_1, m_2) = 1 - \tan\left(\frac{\pi}{8} \sum_{\mathcal{O}_i \subseteq \mathcal{O}} |m_1(\mathcal{O}_i) - m_2(\mathcal{O}_i)|\right)
$$

$$
= 1 - \tan\left(\frac{\pi}{8} \sum_{\mathcal{O}_i \subseteq \mathcal{O}} |m_2(\mathcal{O}_i) - m_1(\mathcal{O}_i)|\right)
$$

$$
= TSM(m_2, m_1)
$$

We can obtain $TSM(m_1, m_2) = TSM(m_2, m_1)$ *.*

Proof 3 For two same BPAs m_1 and m_2 in \mathbb{O} , we have:

$$
TSM(m_1, m_2) = 1 - \tan\left(\frac{\pi}{8} \sum_{\mathcal{O}_i \subseteq \mathcal{O}} |m_1(\mathcal{O}_i) - m_2(\mathcal{O}_i)|\right)
$$

$$
= 1 - \tan\left(\frac{\pi}{8} \sum_{\mathcal{O}_i \subseteq \mathcal{O}} |m_1(\mathcal{O}_i) - m_1(\mathcal{O}_i)|\right)
$$

$$
= 1
$$

Also, suppose that $TSM(m_1, m_2) = 1$, we have:

$$
1 - \tan\left(\frac{\pi}{8} \sum_{\mathcal{O}_i \subseteq \mathbb{O}} |m_1(\mathcal{O}_i) - m_2(\mathcal{O}_i)|\right) = 1
$$

and

$$
\tan\left(\frac{\pi}{8}\sum_{\mathcal{O}_i\subseteq\mathcal{O}}|m_1(\mathcal{O}_i)-m_2(\mathcal{O}_i)|\right)=0
$$

We can get $m_1 = m_2$. Thus, we verify the property of non-degeneracy.

Figure 1: Results of TSM with various λ .

Example 1 *Suppose two BPAs* m_1 *and* m_2 *in* $\mathbb{O} = \{O_1, O_2\}.$

 $m_1: m_1({\mathcal{O}_1}) = \lambda, \quad m_1({\mathcal{O}_2}) = 1 - \lambda$ m_2 : $m_2({\mathcal{O}}_1) = 0.5$, $m_2({\mathcal{O}}_2) = 0.5$

where $0 < \lambda < 1$ *.*

As illustrated in Figure [1,](#page-4-0) where the x-axis indicates variations in λ*, with an interval of 0.05. Specifically, at* $\lambda = 0.5$ *, then* $m_1({\mathcal{O}_1}) = 0.5, m_1({\mathcal{O}_2}) = 0.5$ *, resulting in* m_1 *being equal to* m_2 *, and the* T SM *reaching its maximum value of* 1*. Moreover,* T SM *remains within the range of* [0, 1] *irrespective of changes in* λ *. Additionally, the symmetry of* TSM *is evident as* $TSM(m_1, m_2) = TSM(m_2, m_1)$ *.* This scenario demonstrates the symmetry, bounded, and non-degeneracy properties of TSM .

4. Proposed improved weighted evidence combination method

In this section, a new tangent similarity measure-based evidence combination decision-making method is introduced. Then, We illustrate the effectiveness of the proposed method through a case study of plant disease detection.

4.1 A weighted evidence combination method

Step 1: Let us consider $m_1, m_2, ..., m_n$ as n independent evidences corresponding to the FOD $\mathbb{O} =$ $\{O_1,...O_N\}$. Utilizing the defined tangent similarity measure to calculate the difference between any

Figure 2: The flowchart of the proposed method.

two evidences m_k and m_l , the similarity measure matrix $SMM_{\,n\times n}$ is structured as follows:

$$
SMM_{N\times N} = \begin{bmatrix} 1 & TSM_{12} & \dots & TSM_{1n} \\ TSM_{21} & 1 & \dots & TSM_{2n} \\ \vdots & \ddots & \vdots & \\ TSM_{n1} & TSM_{n2} & \dots & 1 \end{bmatrix}
$$
 (6)

Step 2: For each m_k , compute the support degree Sup_k by summing the similarity of m_k with all other belief functions, represented by:

$$
Sup_k = \sum_{l=1, l \neq k}^{n} TSM_{kl} \tag{7}
$$

Step 3: The weight w_k for each m_k is then calculated based on its support degree relative to the total support degrees of all evidences, expressed as:

$$
w_k = \frac{Sup_k}{\sum_{k=1}^{n} Sup_k}
$$
 (8)

Step 4: Obtain the weighted average evidence \bar{m} as:

$$
\bar{m}(\mathcal{O}_i) = \sum_{k=1}^n w_k \times m_k(\mathcal{O}_i)
$$
\n(9)

Step 5: Utilize Eq. [\(3\)](#page-2-0) to fuse $\bar{m} n - 1$ times.

The flowchart of the proposed method is shown in Figure [2.](#page-5-0)

4.2 Case study in plant disease detection

BPAs	$\{\mathcal{O}_1\}$	$\{\mathcal{O}_2\}$	$\{\mathcal{O}_3\}$	$\{\mathcal{O}_1,\mathcal{O}_2\}$	$\{\mathcal{O}_1,\mathcal{O}_3\}$	\mathbb{O}
m ₁	0.10	0.60	o	O	0.10	0.20
m ₂	O	0.70	O	0.20	O	0.10
m_3	0.8	O	O.1	\circ	O	0.10
m_4	0.20	0.20	0.50	O	0.10	O
m_5	O	0.55	0.20	0.20	O	0.05

Table 1: BPAs modeled in plant disease detection

Plants play a vital role in our ecosystems, supplying essential resources like oxygen and sustenance. Yet, these crucial organisms are vulnerable to diseases that can drastically impact their development and survival. Leaf disease stands out as a prevalent issue that can greatly diminish crop yields and quality, posing significant threats to agricultural productivity and the economic well-being of farmers. Consequently, accurate identification of plant diseases is essential for maintaining robust plant health.

Consider the information from five experts on plant leaf diseases, represented through BPAs. The diseases under consideration include early blight (O_1) , gray leaf spot (O_2) , and bacterial spot (O_3) , which form the framework of discernment $\mathbb{O} = \mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3$. The BPAs from each expert's data are presented in Table [1.](#page-6-0) Notably, most BPAs indicate agreement on disease \mathcal{O}_2 except for m_3 and m_4 . Due to its significant divergence from the collective assessment, m_3 and m_4 are considered unreliable evidence because of their pronounced conflict with the other evidence.

By following the five steps of the combination process to address the plant disease detection prob-lem, the ultimate decision result acquired by fusing all evidences is shown in Table [2](#page-8-0) and Figure [3.](#page-7-0) Notably, when all evidence is combined, our method yields a belief value for the true disease of over 92%. To demonstrate the validity and effectiveness of our combination method, we compare it with the current combination methods. When all evidence is combined, the results of different methods are also presented in Table [2](#page-8-0) and Figure [3.](#page-7-0) The classical Dempster's rule $\lceil 5 \rceil$ struggles to represent truth when evidence conflicts are high. Murphy's method [\[31\]](#page-11-13) fails to recognize the importance of each piece of evidence, making it difficult to achieve higher beliefs in the presence of conflict. Although other methods $[6, 14]$ $[6, 14]$ $[6, 14]$ can also effectively identify disease type, their highest belief degree is lower than the proposed method. Figure [4](#page-9-4) shows the results on identified disease type of different methods. It can be concluded that the proposed method works well, especially in the case of conflicting evidence, and the proposed method has better performance than other methods. The proposed method provides a more accurate view of uncertainty, capturing subtle details that are often ignored by traditional methods. This enhances our understanding of the reliability of information and supports better decision-making.

Figure 3: The results of various methods.

Methods	\tilde{m}	$m_{1,2}$	$m_{1,2,3}$	$m_{1,2,3,4}$	$m_{1,2,3,4,5}$
Dempster's rule [5]	$\{\mathcal{O}_1\}$	0.0581	0.5460	0.6209	0.3711
	$\{\mathcal{O}_2\}$	0.8605	0.4000	0.3095	0.5876
	$\left\{{\cal O}_3\right\}$	$\mathsf O$	0.0162	0.0638	0.0406
	$\{\mathcal{O}_1,\mathcal{O}_2\}$	0.0465	0.0216	$\mathsf O$	$\mathsf O$
	$\{\mathcal{O}_1,\mathcal{O}_3\}$	0.0116	0.0054	0.0058	0.0007
	\mathbb{O}	0.0233	0.0108	$\mathsf O$	$\mathsf O$
Murphy's method [31]	$\overline{\{{\mathcal{O}}_1\}}$	0.0489	0.3526	0.3732	0.1767
	$\{\mathcal{O}_2\}$	0.8592	0.6134	0.5633	0.7967
	$\{\mathcal{O}_3\}$	$\mathsf O$	0.0084	0.0567	0.0245
	$\{\mathcal{O}_1,\mathcal{O}_2\}$	0.0460	0.0140	0.0030	0.0017
	$\{\mathcal{O}_1,\mathcal{O}_3\}$	0.0201	0.0056	0.0030	0.0004
	\mathbb{O}	0.0259	0.0059	0.0007	7×10^{-5}
Deng et al.'s method [6]	$\overline{\{ \mathcal{O}_1 \}}$	0.0489	0.2206	0.2710	0.0760
	$\{\mathcal{O}_2\}$	0.8592	0.7457	0.6696	0.9065
	$\{\mathcal{O}_3\}$	\circ	0.0059	0.0518	0.0154
	$\{\mathcal{O}_1,\mathcal{O}_2\}$	0.0460	0.0146	0.0032	0.0017
	$\{\mathcal{O}_1,\mathcal{O}_3\}$	0.0201	0.0071	0.0036	0.0003
	\mathbb{O}	0.0259	0.0061	0.0008	6×10^{-5}
Lin et al's method $[14]$	$\overline{\{ \mathcal{O}_1 \}}$	0.0489	0.2594	0.2953	0.0996
	$\{\mathcal{O}_2\}$	0.8592	0.7064	0.6365	0.8771
	$\{\mathcal{O}_3\}$	\circ	0.0068	0.0609	0.0212
	$\{\mathcal{O}_1,\mathcal{O}_2\}$	0.0460	0.0143	0.0030	0.0016
	$\{\mathcal{O}_1,\mathcal{O}_3\}$	0.0201	0.0069	0.0035	0.0004
	\mathbb{O}	0.0259	0.0062	0.0008	6×10^{-5}
Proposed method	$\{\mathcal{O}_1\}$	0.0489	0.1515	0.2221	0.0570
	$\{\mathcal{O}_2\}$	0.8592	0.8162	0.7191	0.9268
	$\{\mathcal{O}_3\}$	\circ	0.0041	0.0509	0.0142
	$\{\mathcal{O}_1,\mathcal{O}_2\}$	0.0460	0.0153	0.0031	0.0017
	$\{\mathcal{O}_1,\mathcal{O}_3\}$	0.0201	0.0071	0.0039	0.0003
	$\mathbb O$	0.0259	0.0058	0.0009	6×10^{-5}

Table 2: Results of different methods

5. Conclusions

In this paper, we present a new tangent similarity measure within the DST framework, specifically designed to efficiently handle and resolve conflicts between evidence. Additionally, the paper introduces a new evidence combination technique that leverages the strength of the proposed tangent similarity measure. The proposed method is particularly useful in decision-making environments where managing conflicting evidence poses substantial challenges. The utility and efficacy of the method have been thoroughly demonstrated through a real-world application, confirming not only the effectiveness of the proposed method but also its capacity to enhance decision-making processes. In the future, we hope to extend the proposed method's application to other domains involving uncertainty and imprecision, such as medical diagnosis and autonomous driving systems. In addition, we intend to explore further the potential of tangent similarity measures in identifying differences between ev-

Figure 4: The results on identified disease type (\mathcal{O}_2) of various methods.

idence in the framework of generalized evidence theory.

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Conflicts of Interest

The authors declare no conflicts of interest.

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