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A Novel ARAS-H Approach for Normal T-Spherical Fuzzy Multi-Attribute Group Decision-Making Model with Combined Weights

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ABSTRACT

Compared with the existing fuzzy numbers, normal T-spherical fuzzy numbers (NTSFNs) inherit the merits of T-spherical fuzzy numbers and normal fuzzy numbers. They can describe normal distribution phenomena and neutral information at the same time. It not only has a large expression domain, but also has a strong ability to handle indeterminacy and vagueness. In this article, the main purpose is to introduce a novel distance measure and improve the ARAS (Additive Ratio Assessment) method in the NTSF context to solve the group decision-making problem with combined weight information and make up for the shortcomings of the existing ARAS approaches, for example, correlations between attributes are ignored, the decision process is inflexible, and ARAS has not been extended in the NTSF environment, etc. First, we define a Hamming distance measure with NTSFNs and propose several Aczel-Alsina operational laws of NTSFNs. Then, we develop the normal T-spherical fuzzy Aczel-Alsina Heronian mean (NTSFAAHM) operator and its weighted form (NTSFAAWHM), and related properties and special cases are discussed. Third, For the NTSF multi-attribute group decision-making (MAGDM) problems, we define the NTSF similarity, improve SWARA (Stepwise Weight Assessment Ratio Analysis) and build MDM (maximizing deviation model) to calculate expert weight and attribute subjective and objective weight respectively based on NTSF Hamming distance. Furthermore, we integrate the NTSFAAWHM operator and Hamming distance with ARAS method to form a novel alternative ranking technique, namely NTSF ARAS-H method. Lastly, a numerical example of investment decision for internet waste clothing recycling platform (IWCRP) is presented to show the rationality of methodology. The results indicate that the proposed method has the advantages of clear results, strong operability, reasonable and effective decision-making results, and reliability.

1. Introduction

Considering the vagueness, uncertainty, and incompleteness of evaluation information in MAGDM problems, Atanassov [1] extended an intuitionistic fuzzy set (IFS) with membership degree (MD) (τ) and non-membership degree (ND) (ϑ) based classical fuzzy set (FS) [2]. The Pythagorean fuzzy set (PyFS) was proposed by Yager and Abbasov [3] to make up for the deficiency that $\tau+\vartheta>1$

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($\tau,\vartheta\in[0,1]$), and PyFS satisfies$\tau 2+\vartheta 2$ >1 ($\tau,\vartheta\in[0,1]$). Subsequently, Yager [4] introduced a more flexible q-rung orthopair fuzzy set (q-ROFS) concept, namely flexibly adjusting the decision range expressed by MD and ND through parameter q, and meeting the condition: $\tau q+\vartheta q$ >1 ($\tau,\vartheta\in[0,1]$). However, the evaluation object cannot be fully described by relying only on MD and ND in the above various fuzzy sets. Cuong [5] advanced a picture fuzzy set (PFS) containing MD (τ), abstinence degree (AD) (η), and ND (ϑ) as another form of generalized FS that can describe more information. Although PFS has a stronger ability to describe vagueness and uncertainty than IFS, PyFS, and q-ROFS, it still has no way to deal with the evaluation information when $\tau+\eta+\vartheta>1$ ($\tau,\eta,\vartheta\in[0,1]$). In this regard, the idea of spherical fuzzy set (SFS) was extended and promoted by Mahmood *et al.*[6] to the generalized form, i.e., T-spherical fuzzy set (TSFS). It remove the restriction of decision-makers (DMs) in the allocation of MD, AD, and ND with a larger decision space and enable them to express DMs' preferences and opinions more freely. Obviously, the SFS, PFS, q-ROFS, PyFS, and IFS are all particular examples of TSFS. In addition, there are many phenomena of normal distribution in real life, so it is of great significance to study the integrating of normal distribution. Based on the concepts of normal fuzzy numbers [7] and normal intuitionistic fuzzy numbers [8,9], Liu *et al.* extended an NTSFNs [10,11]. Although the NTSFNs have a large information expression domain and retain the neutral view of DMs, there are few works on NTSFNs, especially the MAGDM method in normal T-spherical fuzzy (NTSF) context. Therefore, the research on NTSF MAGDM method is the first motivation of this paper.

T-norm and T-conorm (TT) are the core foundations of various fuzzy sets operations. Many scholars have studied many aggregation operators based on TT in different fuzzy contexts. For instance, Algebraic TT [12], Einstein TT [13], Hamacher TT [14], Frank TT [15], Dombi TT [16], Schweizer-Sklar TT [17], etc. In 1982, Aczél and Alsina introduced the Aczel-Alsina (AA) TT, which have the advantage of changeability by adjusting a parameter [18]. Hussain *et al.* [19] believed that it is more flexible than the above TT. At present, the research on AA TT has become a hot spot. Many scholars have carried out fruitful study in various fuzzy environments, such as IFS [20], IVIFS[21], PyFS [22], PFS [23,24], hesitant fuzzy set [25], entropy fuzzy element [26], Neutrosophic Z-numbers [27], TSFS [22] and bipolar complex fuzzy [28]. We have noticed that the AA TT operations have not been explored in the NTSF environment. In addition, the relationship between attributes should not be ignored in the actual decision-making situation. Existing aggregation operators such as Bonferroni mean (BM) [29], Heronian mean (HM) [30], Muirhead mean (MM) [31] and Maclaurim symmetric mean (MSM) [32] have the power to identify the correlation among input arguments. Although MSM and MM have the ability to capture multiple input arguments relationships than HM and BM that only consider the correlation of two arguments [33], the formers have higher computational complexity than the latter with the increase of the number of arguments. Moreover, the HM can pay attention to the relationship between input arguments and itself, and reduce computational redundancy, which has more superiorities than BM [34,35]. However, there are no developments of the HM with NTSFNs at present. To sum up, it is necessary to define AA operational laws in NTSF environment and develop it with HM operator. Therefore, this is the second motivation for this work.

ARAS method is an efficient ranking technique, which was introduced by Zavadskas and Turskis [36]. There are five main steps: construction of decision matrix, data normalization, definition of normalized weighting matrix, calculation of optimal function and utility degree, and final ranking of alternatives [37]. The ARAS method simplifies the calculation process of decision-making, and determines the optimal solution of complex decision-making problems through relative index (utility degree). This index can not only remove the impact of various measurement units, but also effectively express the relative difference between the alternative and the ideal solution [37]. Therefore, the merits of ARAS method are summarized as follows: (1) it can be directly proportional to the attribute weight. (2) Able to handle complex decision-making problems. (3) The calculation process is uncomplicated and the result is reliable. In recent years, the ARAS approach has been extended by scholars in different fuzzy decision environments. For example, FSs [38,39], rough sets [40], hesitant linguistic sets [41], neutrosophic sets [42], PFSs[43], IFSs [44], Fermatean fuzzy sets (FFSs) [45], SFSs [46] and q-ROFSs [47], etc. The relevant studies are shown in Table 1.

Table 1
Existing studies on ARAS methods

Authors (Year)	Info. type	Calculation of optimal function			Calculation of utility degree	
		Operational rules	Whether to consider attribute interrelationship	Whether to calculate flexibility	Whether to de fuzzification	Calculation of deviation from ideal solution
Turskis and Zavadskas (2010)[48]	FS	Algebraic	No	No	Yes	Ratio form
Zamani *et al.*(2014)[49]	FS	Algebraic	Yes	No	Yes	Ratio form
Liao *et al.*(2016)[50]	FS	Algebraic	No	No	Yes	Ratio form
Radovic *et al.*(2018) [40]	RS	Algebraic	No	No	No	Ratio form
Dahooie *et al.*(2018)[51]	GS	Algebraic	No	No	Yes	Ratio form
Buyukozkan and Gocer (2018)[52]	IVIFS	Algebraic	No	No	Yes	Ratio form
Iordache *et al.*(2019)[53]	IT2HFS	Algebraic	No	No	Yes	Ratio form
Liao *et al.*(2019)[41]	HFLS	Algebraic	No	No	Yes	Ratio form
Liu *et al.*(2019)[42]	PMVNS	Algebraic	Yes	No	Yes	Ratio form
Buyukozkan and Guler(2020)[76]	HFLS	Algebraic	No	No	Yes	Ratio form
Jovčić *et al.*(2020)[43]	PFS	Algebraic	No	No	Yes	Ratio form
Mishra *et al.*(2020)[44]	IFS	Algebraic	No	No	Yes	Ratio form
Gül (2021)[45]	FFS	Algebraic	No	No	Yes	Ratio form
Gül (2021)[46]	SFS	Algebraic	No	No	Yes	Ratio form
Dorfeshan *et al.*(2021)[54]	IT2FS	Algebraic	No	No	Yes	Ratio form
Karagoz et al.(2021)[55]	IT2FS	Algebraic	No	No	Yes	Ratio form
Mishra and Rani (2021)[47]	q-ROFS	Algebraic	No	No	Yes	Ratio form
Mishra *et al.*(2021)[56]	HFS	Algebraic	No	No	Yes	Ratio form
Mishra *et al.*(2022)[57]	PyFS	Algebraic	No	No	Yes	Ratio form
Rani *et*	PyFS	Algebraic	No	No	Yes	Ratio form

al.(2022)[58]						
Tanackov *et al.*(2022)[59]	IRS	Algebraic	No	No	No	Ratio form
Mishra *et al.*(2023)[60]	IFS	Algebraic	No	No	Yes	Ratio form
Teng and Shen(2023)[61]	UDHLTS	Algebraic	No	No	No	Ratio form
Jaisankar *et al.*(2023)[62]	PHFS	Algebraic	No	No	Yes	Ratio form
Fan *et al.*(2023)[63]	PFS	Algebraic	No	No	Yes	Ratio form
Adah *et al.*(2023)[64]	SVNS	Algebraic	No	No	Yes	Ratio form
This paper	NTSFS	Aczel-Alsina	Yes(NTSFAAWHM)	Yes	No	Hamming distance

Through sorting out the above works, we find that there are four shortcomings in traditional ARAS method. (1) At present, there is no research on extending ARAS method in NTSF context. (2) The interrelationship between input arguments is ignored in the calculation of ARAS. (3) In the existing ARAS methods, the optimal function of each alternative needs to be defuzzified by the score function or expected function before obtaining utility degree. If the expected value of NTSFN proposed by Liu and Wang [11] is used for defuzzification, some information may be lost, because the expected value ignores the impact of standard deviation and AD on the final result of each alternative. (4) The existing researches use the form of ratio for measuring the deviation of the ideal solution, and the premise of this process is to defuzzify the combined values of alternatives. In this form of ratio, if the value of the ideal solution is too large or too small, the final utility degree of the alternatives will be excessively reduced or enlarged, which will result in mistakes in the ranking of the alternatives. Therefore, it is necessary to expand and improve ARAS technique to make up for the above four defects in the NTSF context, which is the third motivation of this paper.

Combining the above three research motivations, the research in this manuscript enriches the normal fuzzy theoretical system, and in the NTSFS environment, we creatively fuse AA and HM to develop novel aggregation operators, and in this way to improve the traditional ARAS method, which is the novel method is more capable of reflecting the actual situation of decision-making problems, and it has a greater practical significance in the practical applications. In summary, the main aim of this article is to extend and improve the ARAS method in the NTSF environment so that it can effectively solve the NTSFMAGDM problem. So, a novel NTSF Hamming distance measure is developed in this article, and the NTSF similarity measure, NTSF MDM-H and NTSF SWARA-H approaches based on this Hamming distance are defined, developed and improved, respectively. Then, the expert weights about the attributes are determined by the NTSF similarity measure, and the objective and subjective weights of the attributes are calculated using the NTSF MDM-H and NTSF SWARA-H approaches, respectively, which leads to the combined weights of the attributes. In addition, we develop the NTSFAAWHM operator for aggregating the evaluation data of each attribute of the alternatives and are able to represent the correlation relationships between attributes and the decision flexibility of the information fusion process. Meanwhile, The NTSF Hamming distance measure is used to substitute the ratio operation to determine the degree of deviation between the combined values of the alternatives and the ideal solution, and then, the prioritization of the alternatives will be determined. The research logic of this paper to solve the NTSFMAGDM problem based on the improved ARAS method is shown in Figure 1.

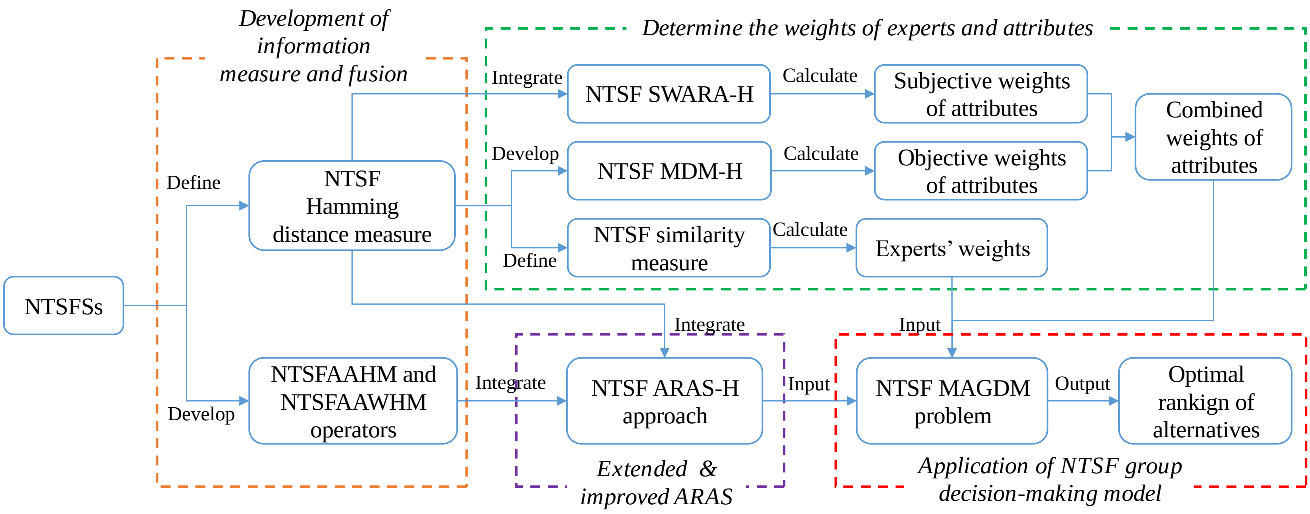

Fig. 1. The research logic of this paper

Some contributions of this article are presented as below.

(1) In this paper, the AA operational laws of NTSFNs are defined, the NTSFAAHM and NTSFAAWHM operators are developed, their related properties and some special cases are discussed.

(2) We define the NTSF Hamming distance. Based on this, the NTSF similarity is defined to determine the expert weight with regard to attributes, the MDM is constructed to calculate the attribute objective weight, and the SWARA is extended to determine the subjective weight of attribute.

(3) The ARAS technique is improved with NTSFNs. The NTSFAAWHM operator is used to capture the correlation of input arguments, and using the defined Hamming distance to measure the deviation between each alternative and the ideal solution.

(4) A numerical example of investment decision for IWCRP is provided to show the feasibility of this method. The reliability, effectiveness and rationality are verified via parameter influence analysis and method comparisons.

The rest of this article is arranged as follows: Some basic notions are reviewed, and some new concepts are defined in Section 2; Section 3 develops the NTSFAAHM and NTSFAAWHM operators. Section 4 designs an ARAS-H-based MAGDM model with NTSFNs. Section 5 gives a numerical example of investment decision of IWCRP to show the proposed method. Section 6 provides the conclusions.

2. Preliminaries

This section briefly reviews the relevant notions, including the NTSFN, AA TT. And we define the Hamming distance measure and develop the AA operational laws of NTSFNs.

2.1 NTSFN and related definitions

Liu and Wang [11] proposed the following definition of NTSFN based on the advantages of TSFS [6] and normal fuzzy number [7].

Definition 1 [11]. Suppose X is a general finite nonempty set, (α,σ) is a normal fuzzy number, $\delta=((\alpha,\sigma), (\tau_\delta, \eta_\delta, \vartheta_\delta))$ is a normal T-spherical fuzzy number (NTSFN). Its MD can be expressed as $\tau_\delta(x)=\tau_\delta \exp\left\{-\left(\frac{x-\alpha}{\sigma}\right)^2\right\}$, $x\in X$. AD is described as $\eta_\delta(x)=1-(1-\eta_\delta)\exp\left\{-\left(\frac{x-\alpha}{\sigma}\right)^2\right\}$, $x\in X$. ND is described as $\vartheta_\delta(x)=1-(1-\vartheta_\delta)\exp\left\{-\left(\frac{x-a}{\sigma}\right)^2\right\}$, $x\in X$. where $0\le\tau_\delta$, η_δ, $\vartheta_\delta\le 1$, they satisfy $0\le\tau_\delta^q+\eta_\delta^q+\vartheta_\delta^q\le 1$, q is a positive integer. And the refusal degree can be described as $\pi_\delta(x)=\sqrt[q]{1-\tau_\delta^q(x)-\eta_\delta^q(x)-\vartheta_\delta^q(x)}$.

Definition 2[11]. Let δ=((α,σ), (τ_δ,η_δ,ϑ_δ)) be a NTSFN, then its expected function ($Ex(\delta)$) can be defined as

$$Ex(\delta)=\frac{\alpha(1+\tau_\delta^q-\vartheta_\delta^q)}{2} \tag{1}$$

Definition 3[11]. For a NTSFN δ=((α,σ), (τ_δ,η_δ,ϑ_δ)), then its score functions ($sc(\delta)$) are defined as

$$sc_1(\delta)=\alpha(\tau_\delta^q-\vartheta_\delta^q),\quad sc_2(\delta)=\sigma(\tau_\delta^q-\vartheta_\delta^q); \tag{2}$$

and accuracy functions ($ac(\delta)$) are described as

$$ac_1(\delta)=\alpha(\tau_\delta^q+\eta_\delta^q+\vartheta_\delta^q);\quad ac_2(\delta)=\sigma(\tau_\delta^q+\eta_\delta^q+\vartheta_\delta^q). \tag{3}$$

For any two NTSFNs δ_1=((α_1,σ_1), (τ_1,η_1,ϑ_1)) and δ_2=((α_2,σ_2), (τ_2,η_2,ϑ_2)), there are the following comparison rules:

(1) If $sc_1(\delta_1)> sc_1(\delta_2)$, then $\delta_1>\delta_2$;

(2) If $sc_1(\delta_1) = sc_1(\delta_2)$ and $ac_1(\delta_1)> ac_1(\delta_2)$, then $\delta_1>\delta_2$;

(3) If $sc_1(\delta_1) = sc_1(\delta_2)$ and $ac_1(\delta_1) = ac_1(\delta_2)$, then

(i) If $sc_2(\delta_1) < sc_2(\delta_2)$, then $\delta_1>\delta_2$;

(ii) If $sc_2(\delta_1) = sc_2(\delta_2)$ and $ac_2(\delta_1) < ac_2(\delta_2)$, then $\delta_1>\delta_2$;

(iii) If $sc_2(\delta_1) = sc_2(\delta_2)$ and $ac_2(\delta_1) = ac_2(\delta_2)$, then $\delta_1=\delta_2$.

Definition 4[11]. Let δ_1=((α_1,σ_1), (τ_1,η_1,ϑ_1)) and δ_2=((α_2,σ_2), (τ_2,η_2,ϑ_2)) be arbitrarily two NTSFNs, real number $\lambda\ge 0$, then their basic operational rules are as following.

(1) $\delta_1\oplus\delta_2=\left((\alpha_1+\alpha_2,\sigma_1+\sigma_2),\left(\sqrt[q]{1-(1-\tau_1^q)(1-\tau_2^q)},\eta_1\eta_2,\vartheta_1\vartheta_2\right)\right)$;

(2) $\delta_1\otimes\delta_2=\left(\left(\alpha_1\alpha_2,\alpha_1\alpha_2\sqrt{\frac{\sigma_1^2}{\alpha_1^2}+\frac{\sigma_2^2}{\alpha_2^2}}\right),\left(\tau_1\tau_2,\sqrt[q]{1-(1-\eta_1^q)(1-\eta_2^q)},\sqrt[q]{1-(1-\vartheta_1^q)(1-\vartheta_2^q)}\right)\right)$;

(3) $\lambda\delta_1=\left((\lambda\alpha_1,\lambda\sigma_1),\left(\sqrt[q]{1-(1-\tau_1^q)^\lambda},\eta_1^\lambda,\vartheta_1^\lambda\right)\right)$;

(4) $\delta_1^\lambda=\left(\left(\alpha_1^\lambda,\lambda^{1/2}\alpha_1^{\lambda-1}\sigma_1\right),\left(\tau_1^\lambda,\sqrt[q]{1-(1-\eta_1^q)^\lambda},\sqrt[q]{1-(1-\vartheta_1^q)^\lambda}\right)\right)$.

Theorem 1 [11]. Let δ_1=((α_1,σ_1), (τ_1,η_1,ϑ_1)) and δ_2=((α_2,σ_2), (τ_2,η_2,ϑ_2)) be arbitrarily two NTSFNs, real numbers λ,λ_1,$\lambda_2\ge 0$, then they satisfy the below operational properties.

(1) $\delta_1\oplus\delta_2=\delta_2\oplus\delta_1$;

(2) $\delta_1\otimes\delta_2=\delta_2\otimes\delta_1$;

(3) $\lambda(\delta_1 \oplus \delta_2)=\lambda\delta_1 \oplus \lambda\delta_2$;

(4) $\lambda_1\delta_1 \oplus \lambda_2\delta_1 = (\lambda_1+\lambda_2)\delta_1$;

(5) $(\delta_1 \otimes \delta_2)^{\lambda}=\delta_1^{\lambda} \otimes \delta_2^{\lambda}$;

(6) $\delta_1^{\lambda_1} \otimes \delta_1^{\lambda_2} = \delta_1^{(\lambda_1+\lambda_2)}$.

Definition 5 [11]**.** Let δ_1=((α_1,σ_1), (τ_1,η_1,ϑ_1)) and δ_2=((α_2,σ_2), (τ_2,η_2,ϑ_2)) be arbitrarily two NTSFNs, $q\geq 1$, then the distance between them is defined as

$$d(\delta_1,\delta_2)=\frac{1}{2}\sqrt{\left(\left(1+\tau_1^q-\eta_1^q-\vartheta_1^q\right)\alpha_1-\left(1+\tau_2^q-\eta_2^q-\vartheta_2^q\right)\alpha_2\right)^2+\frac{1}{2}\left(\left(1+\tau_1^q-\eta_1^q-\vartheta_1^q\right)\sigma_1-\left(1+\tau_2^q-\eta_2^q-\vartheta_2^q\right)\sigma_2\right)^2} \quad (4)$$

The refusal degree of NTSFN is ignored in Eq.(4), which may cause the loss of some evaluation information. To this end, we propose a new Hamming distance measure of NTSFNs, which is defined as below:

Definition 6. Suppose δ_1=((α_1,σ_1), (τ_1,η_1,ϑ_1)) and δ_2=((α_2,σ_2), (τ_2,η_2,ϑ_2)) are arbitrarily two NTSFNs, $q\geq 1$,then their Hamming distance measure can be described as below.

$$D_H(\delta_1,\delta_2)=\begin{pmatrix}\left|\tau_1^q\alpha_1-\tau_2^q\alpha_2\right|+\left|\eta_1^q\alpha_1-\eta_2^q\alpha_2\right|+\left|\vartheta_1^q\alpha_1-\vartheta_2^q\alpha_2\right|+\left|\pi_1^q\alpha_1-\pi_2^q\alpha_2\right|\\+\left|\tau_1^q\sigma_1-\tau_2^q\sigma_2\right|+\left|\eta_1^q\sigma_1-\eta_2^q\sigma_2\right|+\left|\vartheta_1^q\sigma_1-\vartheta_2^q\sigma_2\right|+\left|\pi_1^q\sigma_1-\pi_2^q\sigma_2\right|\end{pmatrix} \quad (5)$$

Theorem 2. Suppose δ_i=((α_i,σ_i), (τ_i,η_i,ϑ_i)) (i=1,2,3) are any three NTSFNs, $q\geq 1$. The Hamming distance of NTSFNs satisfies the below properties.

(1) $D_H(\delta_1,\delta_2)\geq 0$;

(2) If $\delta_1=\delta_2$, then $D_H(\delta_1,\delta_2)=0$;

(3) $D_H(\delta_1,\delta_2)= D_H(\delta_2,\delta_1)$;

(4) $D_H(\delta_1,\delta_2)+ D_H(\delta_2,\delta_3)\geq D_H(\delta_1,\delta_3)$.

Proof: We can easily prove that properties (1)~(3) are all true. Next, we prove property (4). Based on the Definition 1, we have $\alpha_i,\sigma_i>0$, $\tau_i,\eta_i,\vartheta_i\in[0,1]$, and $0\leq\tau_i^q+\eta_i^q+\vartheta_i^q\leq 1$. Then,

$$D_H(\delta_1,\delta_3)=\begin{pmatrix}\left|\tau_1^q\alpha_1-\tau_3^q\alpha_3\right|+\left|\eta_1^q\alpha_1-\eta_3^q\alpha_3\right|+\left|\vartheta_1^q\alpha_1-\vartheta_3^q\alpha_3\right|+\left|\pi_1^q\alpha_1-\pi_3^q\alpha_3\right|\\+\left|\tau_1^q\sigma_1-\tau_3^q\sigma_3\right|+\left|\eta_1^q\sigma_1-\eta_3^q\sigma_3\right|+\left|\vartheta_1^q\sigma_1-\vartheta_3^q\sigma_3\right|+\left|\pi_1^q\sigma_1-\pi_3^q\sigma_3\right|\end{pmatrix}$$

$$=\begin{pmatrix}\left|\tau_1^q\alpha_1-\tau_2^q\alpha_2+\tau_2^q\alpha_2-\tau_3^q\alpha_3\right|+\left|\eta_1^q\alpha_1-\eta_2^q\alpha_2+\eta_2^q\alpha_2-\eta_3^q\alpha_3\right|+\left|\vartheta_1^q\alpha_1-\vartheta_2^q\alpha_2+\vartheta_2^q\alpha_2-\vartheta_3^q\alpha_3\right|+\left|\pi_1^q\alpha_1-\pi_2^q\alpha_2+\pi_2^q\alpha_2-\pi_3^q\alpha_3\right|\\+\left|\tau_1^q\sigma_1-\tau_2^q\sigma_2+\tau_2^q\sigma_2-\tau_3^q\sigma_3\right|+\left|\eta_1^q\sigma_1-\eta_2^q\sigma_2+\eta_2^q\sigma_2-\eta_3^q\sigma_3\right|+\left|\vartheta_1^q\sigma_1-\vartheta_2^q\sigma_2+\vartheta_2^q\sigma_2-\vartheta_3^q\sigma_3\right|+\left|\pi_1^q\sigma_1-\pi_2^q\sigma_2+\pi_2^q\sigma_2-\pi_3^q\sigma_3\right|\end{pmatrix}$$

$$\leq\begin{pmatrix}\left|\tau_1^q\alpha_1-\tau_2^q\alpha_2\right|+\left|\tau_2^q\alpha_2-\tau_3^q\alpha_3\right|+\left|\eta_1^q\alpha_1-\eta_2^q\alpha_2\right|+\left|\eta_2^q\alpha_2-\eta_3^q\alpha_3\right|+\left|\vartheta_1^q\alpha_1-\vartheta_2^q\alpha_2\right|+\left|\vartheta_2^q\alpha_2-\vartheta_3^q\alpha_3\right|+\left|\pi_1^q\alpha_1-\pi_2^q\alpha_2\right|+\left|\pi_2^q\alpha_2-\pi_3^q\alpha_3\right|\\+\left|\tau_1^q\sigma_1-\tau_2^q\sigma_2\right|+\left|\tau_2^q\sigma_2-\tau_3^q\sigma_3\right|+\left|\eta_1^q\sigma_1-\eta_2^q\sigma_2\right|+\left|\eta_2^q\sigma_2-\eta_3^q\sigma_3\right|+\left|\vartheta_1^q\sigma_1-\vartheta_2^q\sigma_2\right|+\left|\vartheta_2^q\sigma_2-\vartheta_3^q\sigma_3\right|+\left|\pi_1^q\sigma_1-\pi_2^q\sigma_2\right|+\left|\pi_2^q\sigma_2-\pi_3^q\sigma_3\right|\end{pmatrix}$$

$$=\begin{pmatrix}\left|\tau_1^q\alpha_1-\tau_2^q\alpha_2\right|+\left|\eta_1^q\alpha_1-\eta_2^q\alpha_2\right|+\left|\vartheta_1^q\alpha_1-\vartheta_2^q\alpha_2\right|+\left|\pi_1^q\alpha_1-\pi_2^q\alpha_2\right|\\+\left|\tau_1^q\sigma_1-\tau_2^q\sigma_2\right|+\left|\eta_1^q\sigma_1-\eta_2^q\sigma_2\right|+\left|\vartheta_1^q\sigma_1-\vartheta_2^q\sigma_2\right|+\left|\pi_1^q\sigma_1-\pi_2^q\sigma_2\right|\end{pmatrix}+\begin{pmatrix}\left|\tau_2^q\alpha_2-\tau_3^q\alpha_3\right|+\left|\eta_2^q\alpha_2-\eta_3^q\alpha_3\right|+\left|\vartheta_2^q\alpha_2-\vartheta_3^q\alpha_3\right|+\left|\pi_2^q\alpha_2-\pi_3^q\alpha_3\right|\\+\left|\tau_2^q\sigma_2-\tau_3^q\sigma_3\right|+\left|\eta_2^q\sigma_2-\eta_3^q\sigma_3\right|+\left|\vartheta_2^q\sigma_2-\vartheta_3^q\sigma_3\right|\left|\pi_2^q\sigma_2-\pi_3^q\sigma_3\right|\end{pmatrix}$$

$$=D_H(\delta_1,\delta_2)+D_H(\delta_2,\delta_3)$$

Thus, $D_H(\delta_1,\delta_2)+ D_H(\delta_2,\delta_3)\geq D_H(\delta_1,\delta_3)$ is hold.

Therefore, the property (4) has been completely proved.

Example 1. Suppose δ_1=((5,5),(0.5,0.3,0.4)), δ_2=((5,5),(0.6,0.0,0.6)), δ_3=((5,5),(0.5,0.5,0.0)) and δ_4=((5,5),(0.7,0.0,0.7)) are four NTSFNs, q=2. Then we calculate the distance values between δ_1 and δ_2, δ_3, δ_4 respectively according to Eq. (4) and Eq.(5), the results are listed in Table 2.

Table 2
Comparison of two distance measures

	$D_H(\delta_1,\delta_2)$	$D_H(\delta_1,\delta_3)$	$D_H(\delta_1,\delta_4)$
Definition 5 (Using Eq.(4))	0.000	0.000	0.000
Definition 6 (Using Eq.(5))	6.200	3.200	11.400

From Table 2, we use the Eq.(4) to calculate the distances between δ_1 and $_2$, δ_3, δ_4 respectively are all zero, while using the Eq. (5) to get different distance values. Obviously, the Eq. (4) ignores the influence of refusal degree in NTSFN on the measurement results, that is, the Eq. (4) cannot reasonably measure the distance between two NTSFNs, which may cause the loss of some evaluation information. Therefore, this comparison in Example 1 shows that the NTSF Hamming distance measure we defined is reasonable.

2.2 Aczel-Alsina operational laws for TSFNs

Definition 7[18]. Suppose x and $y$are real numbers, x, y>0, $\varphi\geq 0$, then the AA TT are described as

$$T_A^{\varphi}(x,y)=\begin{cases} T_D(x,y) & \text{if } \varphi=0 \\ \min(x,y) & \text{if } \varphi\to\infty \\ \exp\left\{-\left((-\ln x)^{\varphi}+(-\ln y)^{\varphi}\right)^{1/\varphi}\right\} & \text{otherwise} \end{cases} \tag{6}$$

$$S_A^{\varphi}(x,y)=\begin{cases} S_D(x,y) & \text{if } \varphi=0 \\ \max(x,y) & \text{if } \varphi\to\infty \\ 1-\exp\left\{-\left((-\ln(1-x))^{\varphi}+(-\ln(1-y))^{\varphi}\right)^{1/\varphi}\right\} & \text{otherwise} \end{cases} \tag{7}$$

Hussain *et al.*[19] proposed AA operational rules of TSFNs based on the AA TT, they are described as below.

Definition 8[19]. Let $\Gamma_1=(\tau_1,\eta_1,\vartheta_1)$ and $\Gamma_2=(\tau_2,\eta_2,\vartheta_2)$ be two TSFNs, λ, $\varphi\geq 0$, the AA operational rules of TSFNs are defined as:

$$(1)\ \Gamma_1\oplus_{AA}\Gamma_2=\left(\begin{array}{l}\sqrt[q]{1-\exp\left\{-\left(\left(-\ln(1-\tau_1^q)\right)^{\varphi}+\left(-\ln(1-\tau_2^q)\right)^{\varphi}\right)^{1/\varphi}\right\}},\sqrt[q]{\exp\left\{-\left(\left(-\ln(\eta_1^q)\right)^{\varphi}+\left(-\ln(\eta_2^q)\right)^{\varphi}\right)^{1/\varphi}\right\}},\\ \sqrt[q]{\exp\left\{-\left(\left(-\ln(\vartheta_1^q)\right)^{\varphi}+\left(-\ln(\vartheta_2^q)\right)^{\varphi}\right)^{1/\varphi}\right\}}\end{array}\right);$$

$$(2)\ \Gamma_1 \otimes_{AA} \Gamma_2 = \left(\begin{array}{l} \sqrt[q]{\exp\left\{-\left(\left(-\ln(\tau_1^q)\right)^{\varphi} + \left(-\ln(\tau_2^q)\right)^{\varphi}\right)^{1/\varphi}\right\}}, \sqrt[q]{1-\exp\left\{-\left(\left(-\ln(1-\eta_1^q)\right)^{\varphi} + \left(-\ln(1-\eta_2^q)\right)^{\varphi}\right)^{1/\varphi}\right\}}, \\ \sqrt[q]{1-\exp\left\{-\left(\left(-\ln(1-\vartheta_1^q)\right)^{\varphi} + \left(-\ln(1-\vartheta_2^q)\right)^{\varphi}\right)^{1/\varphi}\right\}} \end{array} \right);$$

$$(3)\ \lambda\Gamma_1 = \left(\sqrt[q]{1-\exp\left\{-\left(\lambda\left(-\ln(1-\tau_1^q)\right)^{\varphi}\right)^{1/\varphi}\right\}}, \sqrt[q]{\exp\left\{-\left(\lambda\left(-\ln(\eta_1^q)\right)^{\varphi}\right)^{1/\varphi}\right\}}, \sqrt[q]{\exp\left\{-\left(\lambda\left(-\ln(\vartheta_1^q)\right)^{\varphi}\right)^{1/\varphi}\right\}} \right);$$

$$(4)\ (\Gamma_1)^{\lambda} = \left(\sqrt[q]{\exp\left\{-\left(\lambda\left(-\ln(\tau_1^q)\right)^{\varphi}\right)^{1/\varphi}\right\}}, \sqrt[q]{1-\exp\left\{-\left(\lambda\left(-\ln(1-\eta_1^q)\right)^{\varphi}\right)^{1/\varphi}\right\}}, \sqrt[q]{1-\exp\left\{-\left(\lambda\left(-\ln(1-\vartheta_1^q)\right)^{\varphi}\right)^{1/\varphi}\right\}} \right).$$

2.3 AA operational laws of NTSFNs

On the basis of definition 4 and definition 10, the AA operational rules of NTSFNs can be defined as below.

Definition 9. Suppose $\delta_1=((\alpha_1,\sigma_1), (\tau_1,\eta_1,\vartheta_1))$ and $\delta_2=((\alpha_2,\sigma_2), (\tau_2,\eta_2,\vartheta_2))$ are arbitrary two NTSFNs, $\lambda,\varphi\geq 0$. Then they have the following AA operational laws.

$$(1)\ \delta_1 \oplus_{AA} \delta_2 = \left(\begin{pmatrix} \alpha_1+\alpha_2, \\ \sigma_1+\sigma_2 \end{pmatrix}, \left(\begin{array}{l} \sqrt[q]{1-\exp\left\{-\left(\left(-\ln(1-\tau_1^q)\right)^{\varphi} + \left(-\ln(1-\tau_2^q)\right)^{\varphi}\right)^{1/\varphi}\right\}}, \sqrt[q]{\exp\left\{-\left(\left(-\ln(\eta_1^q)\right)^{\varphi} + \left(-\ln(\eta_2^q)\right)^{\varphi}\right)^{1/\varphi}\right\}}, \\ \sqrt[q]{\exp\left\{-\left(\left(-\ln(\vartheta_1^q)\right)^{\varphi} + \left(-\ln(\vartheta_2^q)\right)^{\varphi}\right)^{1/\varphi}\right\}} \end{array} \right) \right);$$

$$(2)\ \delta_1 \otimes_{AA} \delta_2 = \left(\begin{pmatrix} \alpha_1\alpha_2, \\ \alpha_1\alpha_2\sqrt{\frac{\sigma_1^2}{\alpha_1^2}+\frac{\sigma_2^2}{\alpha_2^2}} \end{pmatrix}, \left(\begin{array}{l} \sqrt[q]{\exp\left\{-\left(\left(-\ln(\tau_1^q)\right)^{\varphi} + \left(-\ln(\tau_2^q)\right)^{\varphi}\right)^{1/\varphi}\right\}}, \sqrt[q]{1-\exp\left\{-\left(\left(-\ln(1-\eta_1^q)\right)^{\varphi} + \left(-\ln(1-\eta_2^q)\right)^{\varphi}\right)^{1/\varphi}\right\}}, \\ \sqrt[q]{1-\exp\left\{-\left(\left(-\ln(1-\vartheta_1^q)\right)^{\varphi} + \left(-\ln(1-\vartheta_2^q)\right)^{\varphi}\right)^{1/\varphi}\right\}} \end{array} \right) \right);$$

$$(3)\ \lambda\delta_1 = \left(\begin{pmatrix} \lambda\alpha_1, \\ \lambda\sigma_1 \end{pmatrix}, \left(\sqrt[q]{1-\exp\left\{-\left(\lambda\left(-\ln(1-\tau_1^q)\right)^{\varphi}\right)^{1/\varphi}\right\}}, \sqrt[q]{\exp\left\{-\left(\lambda\left(-\ln(\eta_1^q)\right)^{\varphi}\right)^{1/\varphi}\right\}}, \sqrt[q]{\exp\left\{-\left(\lambda\left(-\ln(\vartheta_1^q)\right)^{\varphi}\right)^{1/\varphi}\right\}} \right) \right);$$

$$(4)\ \delta_1^{\wedge_{AA}\lambda} = \left(\begin{pmatrix} \alpha_1^{\lambda}, \\ \lambda^{1/2}\alpha_1^{\lambda-1}\sigma_1 \end{pmatrix}, \left(\begin{array}{l} \sqrt[q]{\exp\left\{-\left(\lambda\left(-\ln(\tau_1^q)\right)^{\varphi}\right)^{1/\varphi}\right\}}, \sqrt[q]{1-\exp\left\{-\left(\lambda\left(-\ln(1-\eta_1^q)\right)^{\varphi}\right)^{1/\varphi}\right\}}, \\ \sqrt[q]{1-\exp\left\{-\left(\lambda\left(-\ln(1-\vartheta_1^q)\right)^{\varphi}\right)^{1/\varphi}\right\}} \end{array} \right) \right)$$

It is easy to prove that the above calculation results are still NTSFNs, which is omitted.

Theorem 3. Let $\delta_1=((\alpha_1,\sigma_1), (\tau_1,\eta_1,\vartheta_1))$ and $\delta_2=((\alpha_2,\sigma_2), (\tau_2,\eta_2,\vartheta_2))$ be arbitrary two NTSFNs, real numbers λ, λ_1, $\lambda_2\geq 0$, they satisfy the following operational properties.

(1) $\delta_1 \oplus_{AA} \delta_2 = \delta_2 \oplus_{AA} \delta_1$;

(2) $\delta_1 \otimes_{AA} \delta_2 = \delta_2 \otimes_{AA} \delta_1$;

(3) $\lambda(\delta_1 \oplus_{AA} \delta_2) = \lambda\delta_1 \oplus_{AA} \lambda\delta_2$;

(4) $\lambda_1\delta_1 \oplus_{AA} \lambda_2\delta_1 = (\lambda_1 + \lambda_2)\delta_1$;

(5) $(\delta_1 \otimes_{AA} \delta_2)^{\wedge_{AA}\lambda} = \delta_1^{\wedge_{AA}\lambda} \otimes_{AA} \delta_2^{\wedge_{AA}\lambda}$;

(6) $\delta_1^{\lambda_1} \otimes_{AA} \delta_1^{\lambda_2} = \delta_1^{\wedge_{AA}(\lambda_1+\lambda_2)}$.

3. NTSFAAHM aggregation operators

We develop the NTSFAAHM and NTSFAAWHM operators respectively based on the AA operational rules of NTSFNs, and discuss their related properties and some particular cases in this section.

Definition 10[30]**.** For real numbers $x_i \geq 0$ (i=1,2,…,n), s, t >0, the Heronian mean operator is described as

$$HM^{s,t}(x_1, x_2, \ldots, x_n) = \left(\frac{2}{n(n+1)} \sum_{i=1, j=i}^{n} x_i^s x_j^t \right)^{\frac{1}{s+t}} \tag{8}$$

Definition 11. Suppose δ_i=((α_i,σ_i), (τ_i,η_i,ϑ_i)) (i=1,2,…,n) is a family of NTSFNs, then the NTSFAAHM operator can be described as below:

$$NTSFAAHM^{s,t}(\delta_1, \delta_2, \ldots, \delta_n) = \left(\frac{2}{n(n+1)} \mathop{\oplus_{AA}}_{i=1, j=i}^{n} \left((\delta_i)^s \otimes_{AA} (\delta_j)^t \right) \right)^{\frac{1}{s+t}} \tag{9}$$

Theorem 4. For a set of NTSFNs δ_i=((α_i,σ_i), (τ_i,η_i,ϑ_i)) (i=1,2,…,n), the result calculated by the NTSFAAHM operator is still a NTSFN, and even

$$NTSFAAHM^{s,t}(\delta_1,\delta_2,\ldots,\delta_n)=\left(\begin{array}{l}\left(\begin{array}{l}\left(\dfrac{2}{n(n+1)}\displaystyle\sum_{i=1,j=i}^{n}\alpha_i^s\alpha_j^t\right)^{\frac{1}{s+t}},\\ \sqrt{\dfrac{1}{s+t}}\left(\dfrac{2}{n(n+1)}\displaystyle\sum_{i=1,j=i}^{n}\alpha_i^s\alpha_j^t\right)^{\frac{1}{s+t}-1}\cdot\left(\dfrac{2}{n(n+1)}\displaystyle\sum_{i=1,j=i}^{n}\alpha_i^s\alpha_j^t\sqrt{\dfrac{s\sigma_i^2}{\alpha_i^2}+\dfrac{t\sigma_j^2}{\alpha_j^2}}\right)\end{array}\right),\\ \left(\begin{array}{l}\sqrt[q]{\exp\left\{-\left[\dfrac{1}{s+t}\left(-\ln\left(1-\exp\left\{-\left[\dfrac{2T}{n(n+1)}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}},\\ \sqrt[q]{1-\exp\left\{-\left[\dfrac{1}{s+t}\left(-\ln\left(1-\exp\left\{-\left[\dfrac{2N}{n(n+1)}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}},\\ \sqrt[q]{1-\exp\left\{-\left[\dfrac{1}{s+t}\left(-\ln\left(1-\exp\left\{-\left[\dfrac{2V}{n(n+1)}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}}\end{array}\right)\end{array}\right) \tag{10}$$

where

$$T=\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[s\left(-\ln(\tau_i^q)\right)^{\varphi}+t\left(-\ln(\tau_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi},$$

$$N=\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[s\left(-\ln(1-\eta_i^q)\right)^{\varphi}+t\left(-\ln(1-\eta_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi},$$

$$V=\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[s\left(-\ln(1-\vartheta_i^q)\right)^{\varphi}+t\left(-\ln(1-\vartheta_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}.$$

The proof of Theorem 4 refers to **Appendix A**.

According to Theorem 3, we can prove the below three properties of the NTSFAAHM operator:

Theorem 5. Suppose δ_i(i=1,2,…,n) is a set of NTSFNs,

(1) (Idempotency). If $\delta_i=\delta$ for all i, then

$$NTSFAAHM^{s,t}(\delta_1,\delta_2,\ldots,\delta_n)=\delta \tag{11}$$

(2) (Boundedness). If $P^-=\min\{\delta_i\}$, $P^+=\max\{\delta_i\}$, then

$$P^-\leq NTSFAAHM^{s,t}(\delta_1,\delta_2,\ldots,\delta_n)\leq P^+ \tag{12}$$

(3) (Monotonicity). If δ_i^*(i=1, 2,…, n) is also a collection of NTSFNs. For any i, if there is $\delta_i\leq\delta_i^*$, i.e., $\alpha_i\leq\alpha_i^*$, $\tau_i\leq\tau_i^*$, $\eta_i\geq\eta_i^*$ and $\vartheta_i\geq\vartheta_i^*$, then

$$NTSFAAHM^{s,t}(\delta_1,\delta_2,\ldots,\delta_n)\leq NTSFAAHM^{s,t}(\delta_1^*,\delta_2^*,\ldots,\delta_n^*) \tag{13}$$

The proofs of above properties refer to **Appendix B**.

Next, some particular cases of NTSFNs operator are discussed about the parameters s and t.

1) If $t\to 0$, the Eq. (10) is reduced to an NTSF AA generalized mean operator, i.e.,

$$\lim_{t\to 0} NTSFAAHM^{s,t}(\delta_1,\delta_2,\ldots,\delta_n)=\left(\begin{array}{l}\left(\left(\frac{1}{n}\sum_{i=1}^{n}\alpha_i^s\right)^{\frac{1}{s}},\sqrt{\frac{1}{s}}\left(\frac{1}{n}\sum_{i=1}^{n}\alpha_i^s\right)^{\frac{1}{s}-1}\left(\frac{1}{n}\sum_{i=1}^{n}\alpha_i^s\sqrt{\frac{s\sigma_i^2}{\alpha_i^2}}\right)\right),\\ \left(\begin{array}{l}\sqrt[q]{\exp\left\{-\left[\frac{1}{s}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{n}\sum_{i=1}^{n}\left(-\ln\left(1-(\tau_i^q)^{s^{1/\varphi}}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}},\\ \sqrt[q]{1-\exp\left\{-\left[\frac{1}{s}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{n}\sum_{i=1}^{n}\left(-\ln\left(1-(1-\eta_i^q)^{s^{1/\varphi}}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}},\\ \sqrt[q]{1-\exp\left\{-\left[\frac{1}{s}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{n}\sum_{i=1}^{n}\left(-\ln\left(1-(1-\vartheta_i^q)^{s^{1/\varphi}}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}}\end{array}\right)\end{array}\right) \quad (14)$$

2) If s=1, $t\to 0$, the Eq. (10) is reduced to an NTSFAA weight average operator, where weight vector is $w=(1/n,1/n,\ldots,1/n)$ i.e.,

$$\lim_{t\to 0} NTSFAAHM^{1,t}(\delta_1,\delta_2,\ldots,\delta_n)=\left(\left(\begin{array}{l}\frac{1}{n}\sum_{i=1}^{n}\alpha_i,\\ \frac{1}{n}\sum_{i=1}^{n}\sigma\end{array}\right),\left(\begin{array}{l}\sqrt[q]{1-\exp\left\{-\left[\frac{1}{n}\sum_{i=1}^{n}\left(-\ln\left(1-\tau_i^q\right)\right)^{\varphi}\right]^{1/\varphi}\right\}},\\ \sqrt[q]{\exp\left\{-\left[\frac{1}{n}\sum_{i=1}^{n}\left(-\ln\left(\eta_i^q\right)\right)^{\varphi}\right]^{1/\varphi}\right\}},\sqrt[q]{\exp\left\{-\left[\frac{1}{n}\sum_{i=1}^{n}\left(-\ln\left(\eta_i^q\right)\right)^{\varphi}\right]^{1/\varphi}\right\}}\end{array}\right)\right)$$

(15)

3) If s=t=0.5, the Eq. (10) is reduced to an NTSFAA basic HM operator, i.e.,

$$NTSFAAHM^{0.5,0.5}(\delta_1,\delta_2,\ldots,\delta_n)=\left(\begin{array}{l}\left(\dfrac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\alpha_i^{0.5}\alpha_j^{0.5},\dfrac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\alpha_i^{0.5}\alpha_j^{0.5}\sqrt{\dfrac{0.5\sigma_i^2}{\alpha_i^2}+\dfrac{0.5\sigma_j^2}{\alpha_j^2}}\right),\\ \left(\begin{array}{l}\sqrt[q]{1-\exp\left\{-\left[\dfrac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[\begin{array}{l}0.5\left(-\ln(\tau_i^q)\right)^{\varphi}\\+0.5\left(-\ln(\tau_j^q)\right)^{\varphi}\end{array}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}},\\ \sqrt[q]{\exp\left\{-\left[\dfrac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[\begin{array}{l}0.5\left(-\ln(1-\eta_i^q)\right)^{\varphi}\\+0.5\left(-\ln(1-\eta_j^q)\right)^{\varphi}\end{array}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}},\\ \sqrt[q]{\exp\left\{-\left[\dfrac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[\begin{array}{l}0.5\left(-\ln(1-\eta_i^q)\right)^{\varphi}\\+0.5\left(-\ln(1-\eta_j^q)\right)^{\varphi}\end{array}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}}\end{array}\right)\end{array}\right)$$

(16)

4) If s=t=1, the Eq. (10) is reduced to an NTSFAA line HM operator, i.e.,

$$NTSFAAHM^{1,1}(\delta_1,\delta_2,\ldots,\delta_n)=\left(\begin{array}{l}\left(\begin{array}{l}\left(\dfrac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\alpha_i\alpha_j\right)^{\frac{1}{2}},\\ \sqrt{\dfrac{1}{2}}\left(\dfrac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\alpha_i\alpha_j\right)^{-\frac{1}{2}}\left(\dfrac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\alpha_i\alpha_j\sqrt{\dfrac{\sigma_i^2}{\alpha_i^2}+\dfrac{\sigma_j^2}{\alpha_j^2}}\right)\end{array}\right),\\ \left(\begin{array}{l}\sqrt[q]{\exp\left\{-\left[\dfrac{1}{2}\left(-\ln\left(1-\exp\left\{-\left[\dfrac{2T}{n(n+1)}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}},\\ \sqrt[q]{1-\exp\left\{-\left[\dfrac{1}{2}\left(-\ln\left(1-\exp\left\{-\left[\dfrac{2N}{n(n+1)}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}},\\ \sqrt[q]{1-\exp\left\{-\left[\dfrac{1}{2}\left(-\ln\left(1-\exp\left\{-\left[\dfrac{2V}{n(n+1)}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}}\end{array}\right)\end{array}\right) \quad (17)$$

where

$$T=\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[\left(-\ln(\tau_i^q)\right)^{\varphi}+\left(-\ln(\tau_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi},$$

$$N=\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[\left(-\ln(1-\eta_i^q)\right)^{\varphi}+\left(-\ln(1-\eta_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi},$$

$$V=\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[\left(-\ln(1-\eta_i^q)\right)^{\varphi}+\left(-\ln(1-\eta_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}.$$

The NTSFAAHM operator does not consider the importance of input arguments. However, the importance of aggregated arguments plays a significance role in the actual decision-making and evaluation information fusion process. Thus, we should embed weights in the NTSFAAHM operator, and the NTSFAAWHM operator is fined as below:

Definition 12. Let δ_i=((α_i,σ_i), (τ_i,η_i,ϑ_i)) (i=1,2,…,n) be a family of NTSFNs, φ, s, t ≥0. Their weight vector is $w=(w_1,w_2,\ldots,w_n)^T$, with w_i>0 and $\sum_{i=1}^{n} w_i = 1$. If

$$NTSFAAWHM_w^{s,t}(\delta_1,\delta_2,\ldots,\delta_n)=\left(\frac{2}{n(n+1)}\underset{i=1,j=i}{\overset{n}{\oplus_{AA}}}\left((w_i\delta_i)^s\otimes_{AA}(w_j\delta_j)^t\right)\right)^{\frac{1}{s+t}} \tag{18}$$

Theorem 6. Suppose δ_i=((α_i,σ_i), (τ_i,η_i,ϑ_i)) (i=1,2,…,n) is a family of NTSFNs, φ, s, t ≥0. Then according to Eq.(18), the aggregated result is still a NTSFN, and even

$$NTSFAAWHM_w^{s,t}(\delta_1,\delta_2,\ldots,\delta_n)=$$

$$\left(\begin{array}{l}\left(\begin{array}{l}\left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}(w_i\alpha_i)^s(w_j\alpha_j)^t\right)^{\frac{1}{s+t}},\\ \sqrt{\frac{1}{s+t}}\left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}(w_i\alpha_i)^s(w_j\alpha_j)^t\right)^{\frac{1}{s+t}-1}\\ \cdot\left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}(w_i\alpha_i)^s(w_j\alpha_j)^t\sqrt{\frac{s\sigma_i^2}{\alpha_i^2}+\frac{t\sigma_j^2}{\alpha_j^2}}\right)\end{array}\right),\left(\begin{array}{l}\sqrt[q]{\exp\left\{-\left[\frac{1}{s+t}\left(-\ln\left(1-\exp\left\{-\left[\frac{2T^W}{n(n+1)}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}},\\ \sqrt[q]{1-\exp\left\{-\left[\frac{1}{s+t}\left(-\ln\left(1-\exp\left\{-\left[\frac{2N^W}{n(n+1)}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}},\\ \sqrt[q]{1-\exp\left\{-\left[\frac{1}{s+t}\left(-\ln\left(1-\exp\left\{-\left[\frac{2V^W}{n(n+1)}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}}\end{array}\right)\end{array}\right) \tag{19}$$

where

$$T^W=\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[s\left(-\ln\left(1-\exp\left\{-\left[w_i\left(-\ln(1-\tau_i^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}+t\left(-\ln\left(1-\exp\left\{-\left[w_j\left(-\ln(1-\tau_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi},$$

$$N^W=\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[s\left(-\ln\left(1-\exp\left\{-\left[w_i\left(-\ln(\eta_i^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}+t\left(-\ln\left(1-\exp\left\{-\left[w_j\left(-\ln(\eta_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi},$$

$$V^{W}=\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[s\left(-\ln\left(1-\exp\left\{-\left[w_i\left(-\ln(\vartheta_i^{q})\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}+t\left(-\ln\left(1-\exp\left\{-\left[w_j\left(-\ln(\vartheta_j^{q})\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}.$$

Similar to the Theorem 4, the Theorem 6 is true can be proved. We also can prove that the NTSFAAWHM operator has the properties of boundedness and monotonicity, but it has not the property of idempotency.

4. Proposed NTSF MAGDM Model Based on ARAS-H

In this section, we design a MAGDM framework based on the Hamming distance and ARAS-H approach through cohesively integrating the defined Hamming distance, NTSFAAWHM operator and ARAS technique with NTSFNs. This framework is divided into three phases. First, we describe the NTSF group decision-making problem and obtain the assessment data from experts. Then, applying the defined NTSF similarity, the improved SWARA approach and MDM determine the expert weight and attribute subjective and objective weight, respectively. The above methods are developed on the NTSF Hamming distance. Second, the NTSF ARAS-H method improved by the NTSF Hamming distance and NTSFAAWHM operator is employed to rank and select alternatives. The group decision-making framework can be portrayed in Figure 2. The highlighted part in red in this figure is the main focus of this paper.

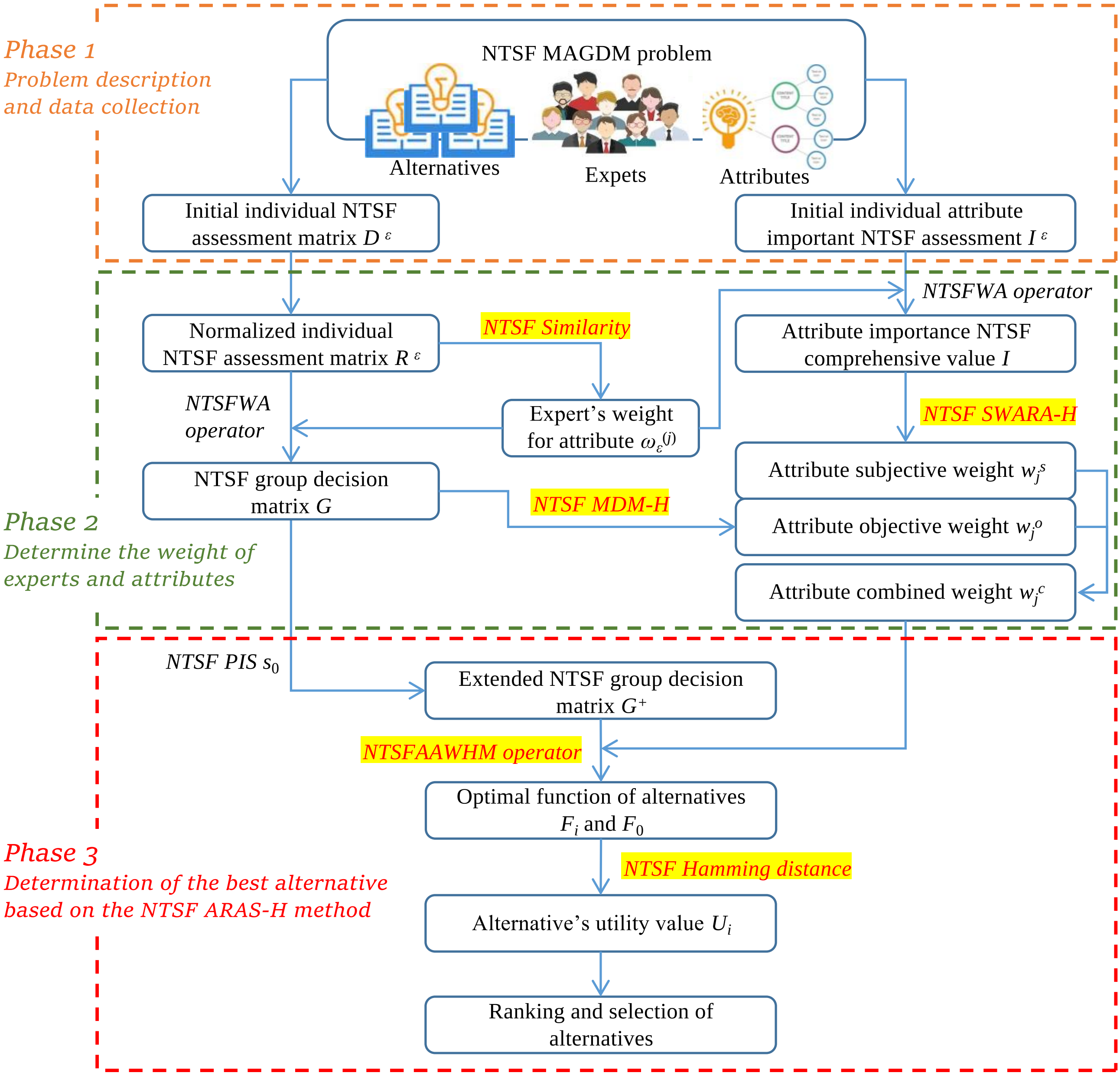

Figure 2. Flowchart of NTSF MAGDM model

Phase 1. Describe Group decision-making problem and collect assessment data

Step 1. We describe the MAGDM problems under NTSF environment as below.

Let a finite alternative set be $H=\{h_1,h_2,\ldots,h_m\}$,an attribute set be $A=\{a_1,a_2,\ldots,a_n\}$, and their weight vector be $w=\{w_1,w_2,\ldots,w_n\}^T$, with $\sum_{j=1}^{n} w_j = 1, w_j \in [0,1]$. Expert set is $E=\{e_1,e_2,\ldots,e_p\}$, who has different knowledge structure, industry background and experience. The experts' evaluation of the alternative is different under a certain attribute, so the allocation of expert weights with regard to attribute is different. Thus, we suppose $\omega_{\varepsilon}^{(j)}$ is a weight of expert e_{ε} corresponding to attribute a_j, with $0 \le \omega_{\varepsilon}^{(j)} \le 1, \sum_{\varepsilon=1}^{p} \omega_{\varepsilon}^{(j)} = 1$. The assessment information is expressed by NTSFNs $d_{ij}^{\varepsilon} = \left((\alpha_{ij}^{\varepsilon},\sigma_{ij}^{\varepsilon}),(\tau_{ij}^{\varepsilon},\eta_{ij}^{\varepsilon},\vartheta_{ij}^{\varepsilon})\right)$, which is provided by expert $e_{\varepsilon}(\varepsilon=1,2,\ldots,p)$ of alternative h_i $(i=1,2,\ldots,m)$ with regard to the attribute $a_j(j=1,2,\ldots,n)$. Thus, we build initial individual NTSF evaluation matrices, i.e., $D^{\varepsilon}=[d_{ij}^{\varepsilon}]_{m\times n}$.

$$
\begin{array}{c c}
 & \begin{array}{cccc} a_1 & a_2 & \cdots & a_n \end{array} \\
D^{\varepsilon} = \begin{array}{c} h_1 \\ h_2 \\ \vdots \\ h_m \end{array} & \begin{bmatrix} d_{11}^{\varepsilon} & d_{12}^{\varepsilon} & \cdots & d_{1n}^{\varepsilon} \\ d_{21}^{\varepsilon} & d_{22}^{\varepsilon} & \cdots & d_{2n}^{\varepsilon} \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1}^{\varepsilon} & d_{m2}^{\varepsilon} & \cdots & d_{mn}^{\varepsilon} \end{bmatrix}
\end{array}
$$

Meanwhile, the importance of attribute is given by experts and expressed it with NTSFNs, and the initial individual attribute importance assessment matrices can be denoted as $I^{\varepsilon}=[\iota_j^{\varepsilon}]$, $\iota_j^{\varepsilon}=\left((\alpha_j^{\varepsilon},\sigma_j^{\varepsilon}),(\tau_j^{\varepsilon},\eta_j^{\varepsilon},\vartheta_j^{\varepsilon})\right)$ (ε=1,2,…,p; j=1,2,…,n), i.e.

$$
\begin{array}{c c}
 & \begin{array}{cccc} a_1 & a_2 & \cdots & a_n \end{array} \\
I^{\varepsilon} = \begin{array}{c} e_1 \\ e_2 \\ \vdots \\ e_p \end{array} & \begin{bmatrix} \iota_1^1 & \iota_2^1 & \cdots & \iota_n^1 \\ \iota_1^2 & \iota_2^2 & \cdots & \iota_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ \iota_1^p & \iota_2^p & \cdots & \iota_n^p \end{bmatrix}
\end{array}
$$

Phase 2. Determine the weight of experts and attributes

Step 2. We get the initial individual NTSF evaluation matrices $D^{\varepsilon}=[d_{ij}^{\varepsilon}]_{m\times n}$ from the experts. The normalized individual NTSF evaluation matrices $R^{\varepsilon}=[r_{ij}^{\varepsilon}]_{m\times n}$ is converted by Eq.(20).

$$
r_{ij}^{\varepsilon}=\begin{cases}\left(\left(\dfrac{\alpha_{ij}^{\varepsilon}}{\max\limits_i\{\alpha_{ij}^{\varepsilon}\}},\dfrac{\sigma_{ij}^{\varepsilon}}{\max\limits_i\{\sigma_{ij}^{\varepsilon}\}}\cdot\dfrac{\sigma_{ij}^{\varepsilon}}{\alpha_{ij}^{\varepsilon}}\right),\left(\tau_{ij}^{\varepsilon},\eta_{ij}^{\varepsilon},\vartheta_{ij}^{\varepsilon}\right)\right), & h_j\in\Psi_1\\ \left(\left(\dfrac{\min\limits_i\{\alpha_{ij}^{\varepsilon}\}}{\alpha_{ij}^{\varepsilon}},\dfrac{\sigma_{ij}^{\varepsilon}}{\max\limits_i\{\sigma_{ij}^{\varepsilon}\}}\cdot\dfrac{\sigma_{ij}^{\varepsilon}}{\alpha_{ij}^{\varepsilon}}\right),\left(\tau_{ij}^{\varepsilon},\eta_{ij}^{\varepsilon},\vartheta_{ij}^{\varepsilon}\right)\right), & h_j\in\Psi_2\end{cases} \tag{20}
$$

whereΨ_1 and $\Psi_2$mean benefit type and cost type attributes respectively.

Step 3. Determine expert weights based on the NTSF similarity

Firstly, we change the individual evaluation matrix D^{ε} into a matrix concerning each attribute, that is, $F^{(j)}=[\xi_{\varepsilon i}^{(j)}]_{p\times m}$ (ε=1,2,…,p, j=1,2,…,n), where $\xi_{\varepsilon i}^{(j)}$ is equal to d_{ij}^{ε}.

The averaging value of alternative h_i about the attribute a_j is $\hat{\xi}_i^{(j)}=\left((\hat{\alpha}_i^{(j)},\hat{\sigma}_i^{(j)}),(\hat{\tau}_i^{(j)},\hat{\eta}_i^{(j)},\hat{\vartheta}_i^{(j)})\right)$ in the matrix $F^{(j)}$.

$$
\hat{\alpha}_i^{(j)}=\frac{1}{p}\sum_{\varepsilon=1}^{p}\alpha_{\varepsilon i}^{(j)},\ \hat{\sigma}_i^{(j)}=\frac{1}{p}\sum_{\varepsilon=1}^{p}\sigma_{\varepsilon i}^{(j)},\ \hat{\tau}_i^{(j)}=\frac{1}{p}\sum_{\varepsilon=1}^{p}\tau_{\varepsilon i}^{(j)},\ \hat{\eta}_i^{(j)}=\frac{1}{p}\sum_{\varepsilon=1}^{p}\eta_{\varepsilon i}^{(j)},\ \hat{\vartheta}_i^{(j)}=\frac{1}{p}\sum_{\varepsilon=1}^{p}\vartheta_{\varepsilon i}^{(j)} \tag{21}
$$

The NTSF similarity measure $sim_{\varepsilon i}^{(j)}$ can be defined between $\xi_{\varepsilon i}^{(j)}$ and $\hat{\xi}_i^{(j)}$ based on the NTSF Hamming distance (see Eq.(5)).

$$
sim_{\varepsilon i}^{(j)}=1-\frac{D_H(\xi_{\varepsilon i}^{(j)},\hat{\xi}_i^{(j)})}{\sum_{\varepsilon=1}^{p}D_H(\xi_{\varepsilon i}^{(j)},\hat{\xi}_i^{(j)})} \tag{22}
$$

The similarity matrix $S^{(j)}$is built by Eq. (22), namely $S^{(j)}=[sim_{\varepsilon i}^{(j)}]_{p\times m}$. Then, we apply the Eq. (23) to obtain the weight of expert e_{ε}concerning the attribute $a_j\in A$.

$$\omega_{\varepsilon}^{(j)} = \frac{\sum_{i=1}^{m} sim_{\varepsilon i}^{(j)}}{\sum_{\varepsilon=1}^{p} \sum_{i=1}^{m} sim_{\varepsilon i}^{(j)}} \tag{23}$$

Obviously, $0 \leq \omega_{\varepsilon}^{(j)} \leq 1, \sum_{\varepsilon=1}^{p} \omega_{\varepsilon}^{(j)} = 1$.

Step 4. The NTSFWA operator is applied to obtain the NTSF group decision matrix $G=[g_{ij}]_{m\times n}$ and attribute importance NTSF comprehensive matrix $I=[Q_j]_{1\times n}$.

The evaluation information of each expert is aggregated by utilizing the NTSFWA operator [11] to obtain the NTSF group decision matrix $G=[g_{ij}]_{m\times n}$, $g_{ij} = \left((\alpha_{ij}, \sigma_{ij}), (\tau_{ij}, \eta_{ij}, \vartheta_{ij})\right)$.

$$g_{ij} = NTSFWA(r_{ij}^1, r_{ij}^2, \ldots, r_{ij}^p) = \left(\left(\sum_{\varepsilon=1}^{p} \omega_{\varepsilon}^{(j)} \alpha_{ij}^{\varepsilon}, \sum_{\varepsilon=1}^{p} \omega_{\varepsilon}^{(j)} \sigma_{ij}^{\varepsilon} \right), \left(\sqrt[q]{1 - \prod_{\varepsilon=1}^{p} \left(1 - (\tau_{ij}^{\varepsilon})^q\right)^{\omega_{\varepsilon}^{(j)}}}, \prod_{\varepsilon=1}^{p} \left(\eta_{ij}^{\varepsilon}\right)^{\omega_{\varepsilon}^{(j)}}, \prod_{\varepsilon=1}^{p} \left(\vartheta_{ij}^{\varepsilon}\right)^{\omega_{\varepsilon}^{(j)}} \right) \right) \tag{24}$$

Similarly, The NTSFWA operator [11] is applied to aggregate the evaluate information of experts to obtain the attribute importance NTSF comprehensive value $Q_j = \left((\alpha_j, \sigma_j), (\tau_j, \eta_j, \vartheta_j)\right)$.

$$Q_j = NTSFWA(\iota_j^1, \iota_j^2, \ldots, \iota_j^p) = \left(\left(\sum_{\varepsilon=1}^{p} \omega_{\varepsilon}^{(j)} \alpha_j^{\varepsilon}, \sum_{\varepsilon=1}^{p} \omega_{\varepsilon}^{(j)} \sigma_j^{\varepsilon} \right), \left(\sqrt[q]{1 - \prod_{\varepsilon=1}^{p} (1 - \tau_j^{\varepsilon})^{\omega_{\varepsilon}^{(j)}}}, \prod_{\varepsilon=1}^{p} (\eta_j^{\varepsilon})^{\omega_{\varepsilon}^{(j)}}, \prod_{\varepsilon=1}^{p} (\vartheta_j^{\varepsilon})^{\omega_{\varepsilon}^{(j)}} \right) \right) \tag{25}$$

Step 5. Determine attribute combination weight.

Step 5.1. Calculate the objective weights by NTSF MDM-H

We employ the MDM technique [65,66] to obtain the objective weight of attributes with completely unknown information. The principle of this technique is that if the deviation between attribute values g_{ij} in attribute a_j of all alternatives is small, then the attribute is assigned a small weight. On the contrary, a larger weight is assigned. In this paper, we embed the Hamming distance into MDM, as follows.

We build the deviation function $D_j(w^o)$ from all alternatives to others with regard to attribute a_j, i.e.,

$$D_j(w^o) = \sum_{i=1}^{m} D_{ij}(w^o) = \sum_{i=1}^{m} \sum_{l=1}^{m} D_H(g_{ij}, g_{lj}) w_j^o \tag{26}$$

where $D_H(g_{ij}, g_{lj})$ is the Hamming distance between NTSFNs g_{ij} and g_{lj}.

So, we get the follow mathematical model

$$\begin{cases} Max\ D(w^o) = \sum_{j=1}^{n} \sum_{i=1}^{m} D_{ij}(w^o) = \sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{l=1}^{m} D_H(g_{ij}, g_{lj}) w_j^o \\ s.t. \quad \sum_{j=1}^{n} (w_j^o)^2 = 1, \quad 0 \leq w_j^o \leq 1 \end{cases} \tag{27}$$

We establish the Lagrange function with the coefficient λ to solve above model. Then we get

$$L(w_j^o, \lambda) = \sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{l=1}^{m} D_H(g_{ij}, g_{lj}) w_j^o + \lambda \left(\sum_{j=1}^{n} (w_j^o)^2 - 1 \right) \tag{28}$$

$$\text{Let}\begin{cases}\dfrac{\partial L(w_j^o,\lambda)}{\partial w_j^o}=\sum_{i=1}^{m}\sum_{l=1}^{m}D_H(g_{ij},g_{lj})w_j^o+2\lambda w_j^o=0\\ \dfrac{\partial L(w_j^o,\lambda)}{\partial\lambda}=\sum_{j=1}^{n}(w_j^o)^2-1=0\end{cases}$$

Then, the optimal objective weight is get as below.

$$w_j^{o*}=\frac{\sum_{i=1}^{m}\sum_{l=1}^{m}D_H(g_{ij},g_{lj})}{\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{l=1}^{m}D_H(g_{ij},g_{lj})\right)^2}}\tag{29}$$

Finally, the objective weight w_j^o of attribute is determined by normalizing w_j^{o*},

$$w_j^o=\frac{\sum_{i=1}^{m}\sum_{l=1}^{m}D_H(g_{ij},g_{lj})}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{l=1}^{m}D_H(g_{ij},g_{lj})}\tag{30}$$

Step 5.2. Calculate the subjective weights by NTSF SWARA-H

The existing SWARA methods [67,68] generally de-fuzzy first, and then arrange them in descending order according to the crisp values, with the difference of the crisp values as the relative importance. And the traditional SWARA approach has not been extended in the NTSFS environment. To eliminate the possibility of partial information loss caused by this process, we use the proposed NTSF Hamming distance measure to calculate the relative importance of attributes. The SWARA is extended with NTSFNs to calculate the subjective weights. The detailed algorithm is as below:

(1) The NTSF importance degrees are arranged in descending order according to NTSFNs comparison rules in Definition 2, and then the relative importance S_j between Q_{j-1} and Q_j is obtained by using NTSF Hamming distance measure (Eq. (5));

$$S_j=D_H(Q_j,Q_{j-1})=\begin{pmatrix}\left|\tau_j^q\alpha_j-\tau_{j-1}^q\alpha_{j-1}\right|+\left|\eta_j^q\alpha_j-\eta_{j-1}^q\alpha_{j-1}\right|+\left|\vartheta_j^q\alpha_j-\vartheta_{j-1}^q\alpha_{j-1}\right|+\left|\pi_j^q\alpha_j-\pi_{j-1}^q\alpha_{j-1}\right|\\+\left|\tau_j^q\sigma_j-\tau_{j-1}^q\sigma_{j-1}\right|+\left|\eta_j^q\sigma_j-\eta_{j-1}^q\sigma_{j-1}\right|+\left|\vartheta_j^q\sigma_j-\vartheta_{j-1}^q\sigma_{j-1}\right|+\left|\pi_j^q\sigma_j-\pi_{j-1}^q\sigma_{j-1}\right|\end{pmatrix}\tag{31}$$

(2) The relative coefficient K_j is determined by Eq.(32)

$$K_j=\begin{cases}1 & j=1\\ S_j+1 & j>1\end{cases}\tag{32}$$

(3) The attribute weight ρ_j is calculated by Eq.(33)

$$\rho_j=\begin{cases}1 & j=1\\ \dfrac{\rho_{j-1}}{K_j} & j>1\end{cases}\tag{33}$$

(4) The subjective weight w_j^s of attribute is determined via normalization (Eq.(34))

$$w_j^s=\frac{\rho_j}{\sum_{j=1}^{n}\rho_j}\tag{34}$$

Step 5.3. Obtain the attribute combined weights.

We use the Eq.(35) to calculate the combined weight w_j^c $(0\le w_j^c\le 1,\sum_{j=1}^{n}w_j^c=1)$

$$w_j^c = \frac{\sqrt{w_j^s w_j^o}}{\sum_{j=1}^{n}\sqrt{w_j^s w_j^o}} \tag{35}$$

So, we can get the attribute combined weight vector $w^c = (w_1^c, w_2^c, \ldots, w_n^c)$.

Phase 3. Ranking alternatives by the NTSF ARAS-H method

Step 6. On the basis of NTSF group decision matrix G, the positive ideal solution $h_0=[g_{0j}]_{1\times n}$ can be obtained by Eq.(36). Therefore, we can get the extended NTSF group decision matrix $G^+ = [g_{ij}]_{(m+1)\times n}$.

$$g_{0j} = \max(g_{ij}) = \left(\left(\max_i \alpha_{ij}, \min_i \sigma_{ij}\right), \left(\max_i \tau_{ij}, \min_i \eta_{ij}, \min_i \vartheta_{ij}\right)\right) \tag{36}$$

$$G^+ = \begin{array}{c} \\ h_0 \\ h_1 \\ h_2 \\ \vdots \\ h_m \end{array} \begin{array}{c} \begin{array}{cccc} a_1 & a_2 & \cdots & a_n \end{array} \\ \begin{bmatrix} g_{01} & g_{02} & \cdots & g_{0n} \\ g_{11} & g_{12} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots & g_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ g_{m1} & g_{m2} & \cdots & g_{mn} \end{bmatrix} \end{array}$$

Step 7. We employ the NTSFAAWHM operator (Eq.(19)) to get the ideal solution optimal function F_0 and the each alternative optimal function F_i (i=1, 2,…,m).

$$\begin{cases} F_0 = NTSFAAWHM_w^{s,t}(g_{01}, g_{02}, \ldots, g_{0n}) = \left((\alpha_0, \sigma_0), (\tau_0, \eta_0, \vartheta_0)\right) \\ F_i = NTSFAAWHM_w^{s,t}(g_{i1}, g_{i2}, \ldots, g_{in}) = \left((\alpha_i, \sigma_i), (\tau_i, \eta_i, \vartheta_i)\right) \end{cases} \tag{37}$$

Step 8. We use the NTSF Hamming distance (Eq.(5)) to obtain the deviation between each alternative and ideal solution, and the utility degree U_i (i=1,2,…,m) of each alternative is calculated by the Eq.(38).

$$U_i = D_H(F_0, F_i) = \begin{pmatrix} \left|\tau_0^q\alpha_0 - \tau_i^q\alpha_i\right| + \left|\eta_0^q\alpha_0 - \eta_i^q\alpha_i\right| + \left|\vartheta_0^q\alpha_0 - \vartheta_i^q\alpha_i\right| + \left|\pi_0^q\alpha_0 - \pi_i^q\alpha_i\right| \\ + \left|\tau_0^q\sigma_0 - \tau_i^q\sigma_i\right| + \left|\eta_0^q\sigma_0 - \eta_i^q\sigma_i\right| + \left|\vartheta_0^q\sigma_0 - \vartheta_i^q\sigma_i\right| + \left|\pi_0^q\sigma_0 - \pi_i^q\sigma_i\right| \end{pmatrix} \tag{38}$$

Step 9. We can rank all the alternatives. The smaller the U_i is, the better the alternative s_i is.

5. Numerical example

In this section, the feasibility of the proposed group decision-making framework is verified by a numerical example of investment decision for IWCRP. Then, the flexibility and effectiveness of the proposed NTSF ARAS-H method are further illustrated by parameter influence analysis and methods comparison study. Also, the comparison with the existing ARAS methods has been implemented, which shows the advantages of the proposed NTSF ARAS-H method for solving complex decision-making problems.

5.1 Investment decision of IWCRP

In 2021, the Ministry of Commerce of China issued the development of China's renewable resources industry. The report showed that recycling networks have been established in most parts of China at present, "Internet + recycling" and other modes have gradually matured, and the renewable resources recycling system integrating recycling, sorting and distribution have gradually improved. In the waste clothing recycling industry, the "Internet +" recycling platform implements the online order and offline recycling mode, leading the transformation and upgrading of the waste clothing recycling industry. At present, many internet recycling platforms have emerged in China's waste clothing recycling market. Due to the fierce competition in the recycling market, the internet recycling platform obtains investment and financing funds from the capital market to expand the market and deepen customer relations, so as to enhance its market competitiveness. However, for investors, how to choose a potential internet recycling platform as an investment object has become a challenging decision-making problem.

LC, a venture capital company, plans to invest in the internet recycling platform for waste clothing. According to the market survey and preliminary screening, there are four Internet recycling platforms (h1, h2, h3, h4) for waste clothing as potential investment objects. In order to screen out the optimal platform project, LC company invited three senior investment experts E={e1,e2,e3}. There are six attributes used to evaluate the alternatives, including: platform operation and maintenance ability (a1), expected revenue (a2), market competitiveness (a3), risk resistance ability (a4), supply chain management ability (a5) and service quality (a6). These attributes are all benefit type.

5.2 Decision process

Step 1. To determine the best internet recycling platform for waste clothing, experts evaluate each platform according to six attributes, and the evaluation values are expressed in NTSFNs. The evaluation results are listed in Tables 3~5, (q=3). Then, experts evaluate the attribute importance and express it with NTSFNs, as shown in Table 6, (q=3).

Table 3
Evaluation information given by e_1

D^1	a_1	a_2	a_3	a_4	a_5	a_6
h_1	((7,5),(0.8,0.4,0.5))	((4,8),(0.7,0.5,0.3))	((9,3),(0.8,0.5,0.5))	((6,6),(0.7,0.6,0.3))	((5,8),(0.6,0.3,0.8))	((8,8),(0.7,0.7,0.3))
h_2	((9,3),(0.6,0.2,0.6))	((6,6),(0.9,0.2,0.4))	((6,4),(0.6,0.3,0.3))	((7,4),(0.8,0.6,0.2))	((7,6),(0.7,0.4,0.4))	((5,7),(0.6,0.1,0.6))
h_3	((4,8),(0.8,0.3,0.1))	((7,6),(0.6,0.2,0.3))	((7,4),(0.8,0.3,0.4))	((7,7),(0.8,0.6,0.4))	((8,5),(0.8,0.5,0.4))	((6,5),(0.8,0.4,0.4))
h_4	((5,5),(0.6,0.5,0.4))	((8,9),(0.7,0.4,0.5))	((7,5),(0.6,0.6,0.3))	((5,4),(0.7,0.5,0.3))	((6,6),(0.7,0.6,0.3))	((7,5),(0.8,0.4,0.5))

Table 4
Evaluation information given by e_2

D^2	a_1	a_2	a_3	a_4	a_5	a_6
h_1	((3,6),(0.6,0.6,0.2))	((5,4),(0.6,0.2,0.3))	((7,6),(0.7,0.3,0.4))	((5,3),(0.5,0.7,0.4))	((6,6),(0.7,0.7,0.3))	((7,4),(0.6,0.5,0.6))
h_2	((7,5),(0.8,0.5,0.5))	((6,7),(0.8,0.4,0.6))	((5,4),(0.6,0.5,0.2))	((6,5),(0.7,0.8,0.2))	((3,8),(0.8,0.1,0.5))	((7,6),(0.8,0.3,0.3))
h_3	((6,4),(0.6,0.2,0.2))	((6,6),(0.7,0.3,0.4))	((8,5),(0.7,0.7,0.3))	((9,3),(0.8,0.5,0.5))	((6,5),(0.9,0.3,0.4))	((8,8),(0.6,0.7,0.1))
h_4	((8,5),(0.7,0.7,0.3))	((7,5),(0.8,0.6,0.3))	((5,6),(0.5,0.5,0.7))	((4,4),(0.7,0.4,0.2))	((4,6),(0.5,0.6,0.5))	((6,6),(0.8,0.2,0.6))

Table 5
Evaluation information given by e_3

D^3	a_1	a_2	a_3	a_4	a_5	a_6

h_1	((4,8),(0.6,0.5, 0.5))	((6,6),(0.6,0.7, 0.4))	((7,4),(0.8,0.3, 0.6))	((8,4),(0.5,0.6, 0.6))	((6,8),(0.7,0.4, 0.7))	((5,8),(0.8,0.4, 0.5))
h_2	((7,3),(0.7,0.3, 0.5))	((7,4),(0.7,0.5, 0.5))	((6,6),(0.7,0.4, 0.6))	((6,3),(0.8,0.4, 0.5))	((5,5),(0.9,0.2, 0.6))	((6,7),(0.8,0.3, 0.5))
h_3	((5,5),(0.9,0.4, 0.3))	((8,7),(0.7,0.1, 0.3))	((9,6),(0.8,0.3, 0.3))	((8,7),(0.8,0.2, 0.2))	((6,6),(0.7,0.5, 0.6))	((8,6),(0.6,0.6, 0.1))
h_4	((6,6),(0.7,0.4, 0.5))	((6,8),(0.7,0.3, 0.6))	((6,4),(0.7,0.1, 0.5))	((5,5),(0.5,0.6, 0.6))	((6,7),(0.6,0.3, 0.4))	((6,5),(0.7,0.4, 0.5))

Table 6
Attribute importance degree given by three experts

E	a_1	a_2	a_3	a_4	a_5	a_6
e_1	((8,6),(0.6,0.7, 0.3))	((7,6),(0.8,0.5, 0.4))	((7,6),(0.7,0.4, 0.3))	((7,7),(0.7,0.4, 0.4))	((6,7),(0.8,0.5, 0.4))	((7,7),(0.6,0.7, 0.3))
e_2	((7,7),(0.6,0.4, 0.4))	((6,7),(0.7,0.4, 0.4))	((6,7),(0.7,0.5, 0.4))	((7,6),(0.6,0.5, 0.2))	((8,6),(0.7,0.4, 0.3))	((7,5),(0.7,0.5, 0.2))
e_3	((7,7),(0.8,0.4, 0.3))	((7,5),(0.7,0.3, 0.3))	((6,7),(0.8,0.4, 0.3))	((8,6),(0.7,0.8, 0.1))	((6,7),(0.7,0.4, 0.4))	((8,7),(0.7,0.5, 0.3))

Step 2. Attributes a_1~a_6 are benefit types. We can get normalized NTSF decision matrix R^{ε}(ε=1, 2, 3) by using Eq. (20), see Tables 7~9.

Table 7
The normalized NTSF decision matrix R^1

R^1	a_1	a_2	a_3	a_4	a_5	a_6
h_1	((0.778,0.625), (0.8,0.4,0.5))	((0.500,0.889), (0.7,0.5,0.3))	((1.000,0.600), (0.8,0.5,0.5))	((0.857,0.857), (0.7,0.6,0.3))	((0.625,1.000), (0.6,0.3,0.8))	((1.000,1.000), (0.7,0.7,0.3))
h_2	((1.000,0.375), (0.6,0.2,0.6))	((0.750,0.667), (0.9,0.2,0.4))	((0.667,0.800), (0.6,0.3,0.3))	((1.000,0.571), (0.8,0.6,0.2))	((0.875,0.750), (0.7,0.4,0.4))	((0.625,0.875), (0.6,0.1,0.6))
h_3	((0.444,1.000), (0.8,0.3,0.1))	((0.875,0.667), (0.6,0.2,0.3))	((0.778,0.8), (0.8,0.3,0.4))	((1.000,1.000), (0.8,0.6,0.4))	((1.000,0.625), (0.8,0.5,0.4))	((0.750,0.625), (0.8,0.4,0.4))
h_4	((0.556,0.625), (0.6,0.5,0.4))	((1.000,1.000), (0.7,0.4,0.5))	((0.778,1.000), (0.6,0.6,0.3))	((0.714,0.571), (0.7,0.5,0.3))	((0.750,0.750), (0.7,0.6,0.3))	((0.875,0.625), (0.8,0.4,0.5))

Table 8
The normalized NTSF decision matrix R^2

R^2	a_1	a_2	a_3	a_4	a_5	a_6
h_1	((0.375,1.000), (0.6,0.6,0.2))	((0.714,0.571), (0.6,0.2,0.3))	((0.875,1.000), (0.7,0.3,0.4))	((0.556,0.600), (0.5,0.7,0.4))	((1.000,0.750), (0.7,0.7,0.3))	((0.875,0.500), (0.6,0.5,0.6))
h_2	((0.875,0.833), (0.8,0.5,0.5))	((0.875,1.000), (0.8,0.4,0.6))	((0.625,0.667), (0.6,0.5,0.2))	((0.667,1.000), (0.7,0.8,0.2))	((0.500,1.000), (0.8,0.1,0.5))	((0.875,0.750), (0.8,0.3,0.3))
h_3	((0.750,0.667), (0.6,0.2,0.2))	((0.875,0.875), (0.7,0.3,0.4))	((1.000,0.833), (0.7,0.7,0.3))	((1.000,0.600), (0.8,0.5,0.5))	((1.000,0.625), (0.9,0.3,0.4))	((1.000,1.000), (0.6,0.7,0.1))
h_4	((1.000,0.833), (0.7,0.7,0.3))	((1.000,0.714), (0.8,0.6,0.3))	((0.625,1.000), (0.5,0.5,0.7))	((0.444,0.800), (0.7,0.4,0.2))	((0.667,0.750), (0.5,0.6,0.5))	((0.750,0.750), (0.8,0.2,0.6))

Table 9
The normalized NTSF decision matrix R^3

R^3	a_1	a_2	a_3	a_4	a_5	a_6

h_1	((0.571,1.000), 0.6,0.5,0.5)	((0.750,0.750), 0.6,0.7,0.4)	((0.778,0.667), 0.8,0.3,0.6)	((1.000,0.571), 0.5,0.6,0.6)	((1.000,1.000), 0.7,0.4,0.7)	((0.625,1.000), 0.8,0.4,0.5)
h_2	((1.000,0.375), 0.7,0.3,0.5)	((0.875,0.500), 0.7,0.5,0.5)	((0.667,1.000), 0.7,0.4,0.6)	((0.750,0.429), 0.8,0.4,0.5)	((0.833,0.625), 0.9,0.2,0.6)	((0.750,0.875), 0.8,0.3,0.5)
h_3	((0.714,0.625), 0.9,0.4,0.3)	((1.000,0.875), 0.7,0.1,0.3)	((1.000,1.000), 0.8,0.3,0.3)	((1.000,1.000), 0.8,0.2,0.2)	((1.000,0.750), 0.7,0.5,0.6)	((1.000,0.750), 0.6,0.6,0.1)
h_4	((0.857,0.750), 0.7,0.4,0.5)	((0.750,1.000), 0.7,0.3,0.6)	((0.667,0.667), 0.7,0.1,0.5)	((0.625,0.714), 0.5,0.6,0.6)	((1.000,0.875), 0.6,0.3,0.4)	((0.750,0.625), 0.7,0.4,0.5)

Step 3: We calculate the expert weight with regard to attributes. Take attribute h_1 as an example, we have

$$F^{(1)}=\begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \end{array}\begin{array}{c} \begin{array}{cccc} h_1 & h_2 & h_3 & h_4 \end{array} \\ \begin{bmatrix} ((0.778,0.625),(0.8,0.4,0.5)) & ((1.000,0.375),(0.6,0.2,0.6)) & ((0.444,1.000),(0.8,0.3,0.1)) & ((0.556,0.625),(0.6,0.5,0.4)) \\ ((0.375,1.000),(0.6,0.6,0.2)) & ((0.875,0.833),(0.8,0.5,0.5)) & ((0.750,0.667),(0.6,0.2,0.2)) & ((1.000,0.833),(0.7,0.7,0.3)) \\ ((0.571,1.000),(0.6,0.5,0.5)) & ((1.000,0.375),(0.7,0.3,0.5)) & ((0.714,0.625),(0.9,0.4,0.3)) & ((0.857,0.750),(0.7,0.4,0.5)) \end{bmatrix} \end{array}$$

Then the evaluation mean value of attribute h_1 is calculated by the Eq.(21).

$\hat{\xi}_1^{(1)}=\big((0.575,0.875),(0.667,0.500,0.400)\big)$, $\hat{\xi}_2^{(1)}=\big((0.958,0.528),(0.700,0.333,0.533)\big)$,

$\hat{\xi}_3^{(1)}=\big((0.636,0.764),(0.767,0.300,0.200)\big)$, $\hat{\xi}_4^{(1)}=\big((0.804,0.736),(0.667,0.533,0.400)\big)$.

Furthermore, the similarity matrix $S^{(1)}$ can be obtained by the Eq. (22).

$$S^{(1)}=\begin{bmatrix} 0.512 & 0.702 & 0.787 & 0.755 \\ 0.675 & 0.455 & 0.649 & 0.464 \\ 0.813 & 0.843 & 0.564 & 0.781 \end{bmatrix}$$

From Eq. (23), the weight value of experts with regard to attribute h_1 is $\omega_1^{(1)}=0.345,\omega_2^{(1)}=0.280,\omega_3^{(1)}=0.375$ .Similarly, we get

$\omega_1^{(2)}=0.362,\omega_2^{(2)}=0.315,\omega_3^{(2)}=0.322$; $\omega_1^{(3)}=0.375,\omega_2^{(3)}=0.291,\omega_3^{(3)}=0.334$;

$\omega_1^{(4)}=0.361,\omega_2^{(4)}=0.322,\omega_3^{(4)}=0.318$;

$\omega_1^{(5)}=0.347,\omega_2^{(5)}=0.328,\omega_3^{(5)}=0.325$; $\omega_1^{(6)}=0.310,\omega_2^{(6)}=0.340,\omega_3^{(6)}=0.350$.

Step 4. We apply the NTSFWA operator (Eq. (24)) to obtain NTSF group decision matrix G, which can be shown in Table 10. Also, we can obtain the attribute importance NTSF comprehensive value by Eq.(25).

Q_1=((7.345,6.655),(0.700,0.485,0.325)), Q_2=((6.685,5.993),(0.743,0.395,0.365)),
Q_3=((6.375,6.625),(0.740,0.427,0.326)), Q_4=((7.318,6.361),(0.673,0.536,0.206)),
Q_5=((6.656,6.672),(0.741,0.432,0.364)), Q_6=((7.350,6.321),(0.674,0.555,0.261)).

Table 10
NTSF group decision matrix G

G	a_1	a_2	a_3
h_1	((0.587,0.871),(0.694,0.487,0.387))	((0.648,0.744),(0.642,0.417,0.329))	((0.889,0.739),(0.776,0.363,0.498))
h_2	((0.965,0.504), (0.710,0.301,0.532))	((0.824,0.718), (0.827,0.334,0.188))	((0.655,0.828), (0.639,0.383,0.336))
h_3	((0.631,0.766), (0.821,0.298,0.183))	((0.910,0.794), (0.669,0.182,0.328))	((0.917,0.876), (0.876,0.384,0.334))
h_4	((0.793,0.730),	((0.919,0.910),	((0.696,0.889),

	(0.671,0.505,0.401))	(0.738,0.414,0.451))	(0.619,0.313,0.455))
	a_4	a_5	a_6
h_1	((0.806,0.684),(0.595,0.684,0.595))	((0.870,0.918),(0.671,0.435,0.555))	((0.826,0.830),(0.719,0.513,0.454))
h_2	((0.813,0.664), (0.774,0.579,0.268))	((0.739,0.791), (0.821,0.203,0.491))	((0.754,0.833), (0.757,0.213,0.445))
h_3	((1.000,0.871), (0.800,0.399,0.345))	((1.000,0.666), (0.823,0.423,0.456))	((0.922,0.796), (0.686,0.557,0.154))
h_4	((0.599,0.690), (0.654,0.493,0.328))	((0.804,0.791), (0.618,0.479,0.390))	((0.789,0.667), (0.771,0.316,0.532))

Step 5. We determine the attribute combined weight. First, the attribute objective weight vector is determined by Eqs. (26~30) as w^o=(0.156,0.188,0.153,0.171,0.203, 0.129).

Then, we compute and obtain the attribute subjective weight vector via the Eqs. (31~34), i.e., w^s=(0.057,0.494,0.165,0.021,0.252,0.012).

Finally, the attribute combined weight vector is calculated by Eq. (35), i.e., w^c=(0.106,0.345,0.180,0.068,0.256,0.044).

Step 6. According to the group decision matrix *G* (in Table 9), the positive ideal solution h_0 can be obtained by Eq. (36). Then the extended NTSF group decision matrix is shown in Table 11.

h_0={((0.965, 0.504), (0.821,0.298,0.183); ((0.919, 0.718), (0.827,0.182,0.328)); ((0.917, 0.739), (0.776,0.313,0.334)); ((1.000, 0.664), (0.800,0.339,0.268)); ((1.000, 0.666), (0.823,0.203,0.390)); ((0.922, 0.667), (0.771,0.213,0.154))}.

Table 11
The extended NTSF group decision matrix *G+*

G	a_1	a_2	a_3
h_0	((0.965,0.504), (0.821,0.298,0.183))	((0.919,0.718), (0.827,0.182,0.328))	((0.917,0.739), (0.776,0.313,0.334))
h_1	((0.587,0.871), (0.694,0.487,0.387))	((0.648,0.744), (0.642,0.417,0.329))	((0.889,0.739), (0.776,0.363,0.498))
h_2	((0.965,0.504), (0.710,0.301,0.532))	((0.824,0.718), (0.827,0.334,0.188))	((0.655,0.828), (0.639,0.383,0.336))
h_3	((0.631,0.766), (0.821,0.298,0.183))	((0.910,0.794), (0.669,0.182,0.328))	((0.917,0.876), (0.876,0.384,0.334))
h_4	((0.793,0.730), (0.671,0.505,0.401))	((0.919,0.910), (0.738,0.414,0.451))	((0.696,0.889), (0.619,0.313,0.455))
	a_4	a_5	a_6
h_0	((1.000,0.664), (0.800,0.339,0.268))	((1.000,0.666), (0.823,0.203,0.390))	((0.922,0.667), (0.771,0.213,0.154))
h_1	((0.806,0.684), (0.595,0.684,0.595))	((0.870,0.918), (0.671,0.435,0.555))	((0.826,0.830), (0.719,0.513,0.454))
h_2	((0.813,0.664), (0.774,0.579,0.268))	((0.739,0.791), (0.821,0.203,0.491))	((0.754,0.833), (0.757,0.213,0.445))
h_3	((1.000,0.871), (0.800,0.399,0.345))	((1.000,0.666), (0.823,0.423,0.456))	((0.922,0.796), (0.686,0.557,0.154))
h_4	((0.599,0.690), (0.654,0.493,0.328))	((0.804,0.791), (0.618,0.479,0.390))	((0.789,0.667), (0.771,0.316,0.532))

Step 7. We employ the NTSFAAWHM operator (Eq.(37)) to aggregate the attribute variables of each alternative to obtain the following results.(φ=2, s=t=1).

F_0=((0.770,0.550),(0.763,0.400,0.386)), F_1=((0.617,0.656),(0.649,0.563,0.561)), F_2=((0.633,0.606),(0.719,0.450,0.528)), F_3=((0.734,0.640),(0.727,0.500,0.413)), F_4=((0.645,0.669),(0.635,0.530,0.525)).

Step 8. We use the Eq.(38) to calculate the utility degree of each alternative.
U_1=0.640, U_2=0.350, U_3=0.225, U_4=0.581.

Step 9. According to the utility degree of alternative, the ranking result is$h_3 \succ h_2 \succ h_4 \succ h_1$, where the symbol "≻" means "superior to". Therefore, the alternative h_3 is the best investment object of internet recycling platform for waste clothing.

5.3 Analysis and discussions

5.3.1 Parameters influence analysis

In the above case, we set the parameters in the proposed method as q=3, φ=2, s=t=1to get the final result. It is necessary to discuss whether the final ranking of the alternative in this real case will be affected with different parameter values.

Firstly, the alternative ranking is discussed by taking different values of parameter φ in AA TT. During the computational process of the NTSFAAWHM operator in Step 7, we selected various values from the parameter $\varphi \in [1,100]$. The final results of each alternative are shown in Table 12.

Table 12
The final results of alternatives with different φ

φ	Results	Rankings
1	U_1=0.675, U_2=0.366, U_3=0.232, U_4=0.566	$h_3 \succ h_2 \succ h_4 \succ h_1$
2	U_1=0.640, U_2=0.350, U_3=0.225, U_4=0.581	$h_3 \succ h_2 \succ h_4 \succ h_1$
5	U_1=0.611, U_2=0.305, U_3=0.204, U_4=0.548	$h_3 \succ h_2 \succ h_4 \succ h_1$
10	U_1=0.597, U_2=0.273, U_3=0.187, U_4=0.500	$h_3 \succ h_2 \succ h_4 \succ h_1$
20	U_1=0.581, U_2=0.254, U_3=0.172, U_4=0.454	$h_3 \succ h_2 \succ h_4 \succ h_1$
50	U_1=0.551, U_2=0.240, U_3=0.159, U_4=0.428	$h_3 \succ h_2 \succ h_4 \succ h_1$
100	U_1=0.537, U_2=0.235, U_3=0.156, U_4=0.421	$h_3 \succ h_2 \succ h_4 \succ h_1$

The results in Table 12 show that the final results of the alternatives decrease as the parameter φincreases on the whole. When the parameter $\varphi \in [1,100]$ takes different values, we get the utility degree of each alternative, and the final ranking results of the alternatives are consistent, that is, $h_3 \succ h_2 \succ h_4 \succ h_1$. Therefore, the proposed method is not sensitive to change of parameterφ.

Then, we discuss the influence of different values of parameters s and t on the final ranking of the alternative. In order to reflect the influence of parameters s, t and the interrelationship between these attributes, we take s and t values into three types, namely $s>t$, $s=t$ and $s<t$. We applied this to the calculation of the NTSFAAWHM operator in Step 7. The final results of each alternative are shown in Table 13, and the ranking change of each alternative is shown in Figure 3.

From Table 13 and Fig. 3, when parameters s=0, t=1 and s=1, t=0 indicate that there is no correlation between attributes, and the alternative ranking of both conditions is $h_3 \succ h_2 \succ h_4 \succ h_1$. As the values of s and t increase, the degree of association between attributes increases, and the alternative ranking under different values is $h_3 \succ h_2 \succ h_4 \succ h_1$, and the ranking is stable. Therefore, this means that the parameters s, t are not sensitive to the final result of the alternative.

Table 13
The final results of alternatives with different s and t

s, t	Results	Rankings
0,1	U_1=0.471, U_2=0.236, U_3=0.126, U_4=0.440	$h_3 \succ h_2 \succ h_4 \succ h_1$
1,0	U_1=0.556, U_2=0.324, U_3=0.266, U_4=0.528	$h_3 \succ h_2 \succ h_4 \succ h_1$
0.5,0.5	U_1=0.509, U_2=0.278, U_3=0.194, U_4=0.452	$h_3 \succ h_2 \succ h_4 \succ h_1$
1,1	U_1=0.640, U_2=0.350, U_3=0.225, U_4=0.581	$h_3 \succ h_2 \succ h_4 \succ h_1$

1,3	U_1=0.813, U_2=0.414, U_3=0.236, U_4=0.741	$h_3 \succ h_2 \succ h_4 \succ h_1$
3,1	U_1=0.839, U_2=0.475, U_3=0.324, U_4=0.743	$h_3 \succ h_2 \succ h_4 \succ h_1$
3,3	U_1=0.881, U_2=0.480, U_3=0.269, U_4=0.713	$h_3 \succ h_2 \succ h_4 \succ h_1$
3,5	U_1=0.934, U_2=0.493, U_3=0.261, U_4=0.869	$h_3 \succ h_2 \succ h_4 \succ h_1$
5,3	U_1=0.981, U_2=0.532, U_3=0.341, U_4=0.888	$h_3 \succ h_2 \succ h_4 \succ h_1$
5,5	U_1=1.004, U_2=0.531, U_3=0.294, U_4=0.942	$h_3 \succ h_2 \succ h_4 \succ h_1$
7,7	U_1=1.077, U_2=0.557, U_3=0.324, U_4=1.038	$h_3 \succ h_2 \succ h_4 \succ h_1$
9,9	U_1=1.121, U_2=0.587, U_3=0.348, U_4=1.104	$h_3 \succ h_2 \succ h_4 \succ h_1$

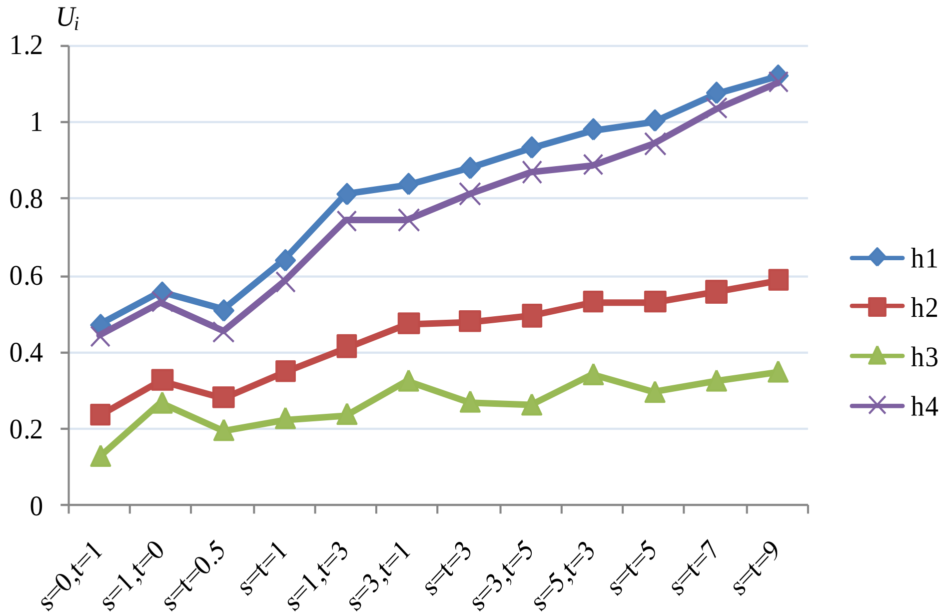

Figure 3. The utility degrees of alternatives with different *s* and *t*

For parameter q, we change its value to observe its impact on alternative ranking. We take different values in the $q\in[3,21]$. The final results of the alternative are shown in Table 14. The ranking of the alternatives is shown in Figure 4.

From Table 14 and Figure 4, the final results of the alternatives decrease with the increase of parameter q on the whole. When different values are taken for $q\in[3,21]$, the final ranking of the alternatives is $h_3 \succ h_2 \succ h_4 \succ h_1$, and the optimal option is always h_3. Therefore, the parameter q in the proposed method has no effect on the final ranking of the alternative, and has stability and reliability.

Table 14
The final results of alternatives with different q

q	Results	Rankings
3	U_1=0.640, U_2=0.350, U_3=0.225, U_4=0.581	$h_3 \succ h_2 \succ h_4 \succ h_1$
5	U_1=0.523, U_2=0.284, U_3=0.185, U_4=0.480	$h_3 \succ h_2 \succ h_4 \succ h_1$
7	U_1=0.439, U_2=0.240, U_3=0.169, U_4=0.405	$h_3 \succ h_2 \succ h_4 \succ h_1$
9	U_1=0.392, U_2=0.219, U_3=0.163, U_4=0.361	$h_3 \succ h_2 \succ h_4 \succ h_1$
11	U_1=0.365, U_2=0.208, U_3=0.161, U_4=0.334	$h_3 \succ h_2 \succ h_4 \succ h_1$
13	U_1=0.346, U_2=0.201, U_3=0.159, U_4=0.314	$h_3 \succ h_2 \succ h_4 \succ h_1$
15	U_1=0.333, U_2=0.196, U_3=0.157, U_4=0.300	$h_3 \succ h_2 \succ h_4 \succ h_1$
17	U_1=0.323, U_2=0.192, U_3=0.156, U_4=0.290	$h_3 \succ h_2 \succ h_4 \succ h_1$
19	U_1=0.316, U_2=0.190, U_3=0.155, U_4=0.282	$h_3 \succ h_2 \succ h_4 \succ h_1$
21	U_1=0.310, U_2=0.188, U_3=0.153, U_4=0.276	$h_3 \succ h_2 \succ h_4 \succ h_1$

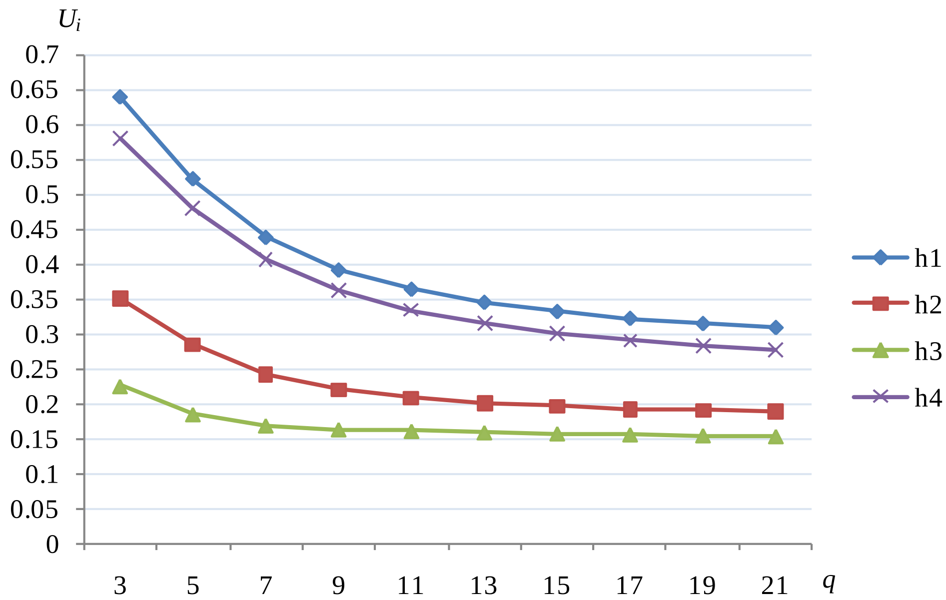

Figure 4. The utility degrees of alternatives with different q

5.3.2 Comparative studies

(1) Comparison with aggregation operators and ranking techniques

To verify the effectiveness of our method, the comparison with the existing MCDM techniques including aggregation operator and ranking method (such as SpNoFWBM [69], SNoFWPMM [70], NTSFWA [10], NTSFWG[10], NTSFTODIM [10], NTSF Taxonomy [10], NTSFWMSM [11] is performed. These methods are applied to solve the above case, and the results are shown in table 15.

Table 15
The results obtained by different methods

Methods	Results	Rankings
SpNoFWBM [69]	Cannot be calculated	No
SNoFWPMM [70]	Cannot be calculated	No
NTSFWA [10]	$Ex(s_1)$=0.474, $Ex(s_2)$=0.548, $Ex(s_3)$=0.644, $Ex(s_4)$=0.502	$h_3 \succ h_2 \succ h_4 \succ h_1$
NTSFWG [10]	$Ex(s_1)$=0.457, $Ex(s_2)$=0.527, $Ex(s_3)$=0.622, $Ex(s_4)$=0.491	$h_3 \succ h_2 \succ h_4 \succ h_1$
NTSFWMSM [11]	$Ex(s_1)$=0.874, $Ex(s_2)$=0.898, $Ex(s_3)$=0.926, $Ex(s_4)$=0.880	$h_3 \succ h_2 \succ h_4 \succ h_1$
NTSFTODIM [10]	ξ_1=0.000, ξ_2=0.565, ξ_3=1.000, ξ_4=0.238	$h_3 \succ h_2 \succ h_4 \succ h_1$
NTSFTaxonomy [10]	F_1=0.210, F_2=0.479, F_3=0.796, F_4=0.303	$h_3 \succ h_2 \succ h_4 \succ h_1$
This paper	U_1=0.640, U_2=0.350, U_3=0.225, U_4=0.581	$h_3 \succ h_2 \succ h_4 \succ h_1$

Table 15 shows that the SpNoFWBM and SNoFWPMM operators cannot be applied to this case, and the ranking results of each alternative cannot be obtained. Obviously, spherical normal fuzzy number is a special case of NTSFN, so the proposed method is more general and flexible. From the decision results, the proposed method and other methods get the same ranking, that is, $h_3 \succ h_2 \succ h_4 \succ h_1$, and the best alternative is h_3. This can explain the effectiveness and practicality of our method. However, in terms of decision-making process, there are some differences between this method and the above method. Specifically, (1) compared with the NTSFWA, NTSFWG and

NTSFWMSM operators, the NTSFAAWHM operator in the proposed method can make up for the fact that the NTSFWA and NTSFWG operators cannot capture the interrelationship between attributes. Although the NTSFWMSM operator can capture the interrelationship of multiple attributes, its calculation process is more complex than the NTSFAAWHM operator. In addition, there are fewer adjustable parameters in the existing aggregation operator than the NTSFAAWHM operator, which shows that the decision-making flexibility of our method is high. (2) Compared with the NTSFTODIM and NTSF Taxonomy methods. The NTSFTODIM method calculates the dominance degrees of each alternative with regard to attributes. If the number of attributes or alternatives is large, then the complexity of calculation increases. Similarity, the decision-making steps in the NTSF Taxonomy method are more and the calculation process is more complex than our NTSFARAS method. (3) In addition, both NTSFTODIM and NTSF Taxonomy adopt distance measures without refusal degree, which may cause partial information loss in the decision-making process. In this regard, the NTSF Hamming distance we defined can make up for this shortcoming. In a word, the above comparative analysis can show that the method proposed in this paper is more reasonable than the existing methods.

(2) Comparison with existing ARAS methods

Next, we compare the proposed NTSF ARAS-H method with some existing ARAS methods. These existing ARAS methods were extended in IFSs [44], FFSs [45], *q*-ROFSs [47], PFSs [43] and SFSs [46]. In terms of decision-making process, these methods are compared with the method proposed in this paper, as shown in Figure 5.

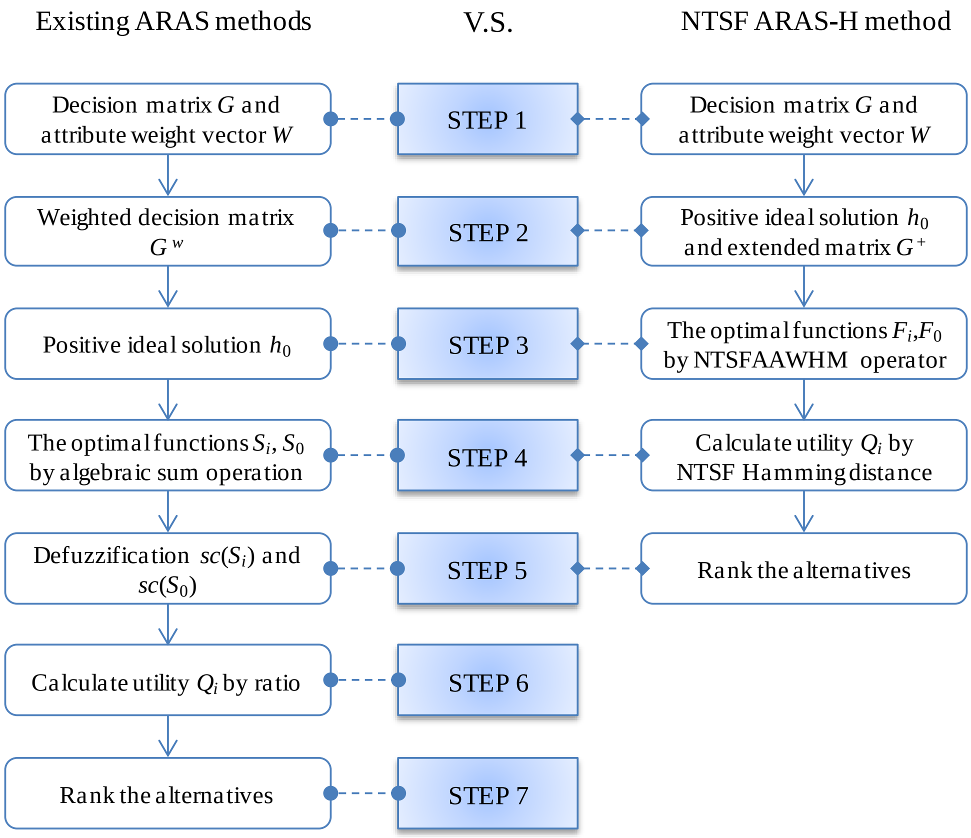

Figure 5. Comparison on decision-making process

From Figure 5, the steps of the NTSF ARAS-H method are fewer and simpler. In terms of aggregation of evaluation values for different attributes of each alternative, these existing methods often apply algebraic sum operation to fuse evaluation values, so as to obtain the optimal function of each alternative. However, the interrelationship between attributes is ignored in this process, which is not consistent with the actual decision-making problems. In terms of defuzzification, Mishra *et al.*[44] and Mishra and Rani[47] adopted different fuzzy score functions to precision after the weighting matrix stage, and Gül [45,46] used the score functions to de fuzzy before the stage of utility degree of alternative by ratio, Jovčić *et al.*[43] employed the two-step defizzification method for defuzzification before using the ratio to obtain the utility degree of the alternative. Although these methods have slightly different positions in the defuzzification stage of the decision-making process, their purpose is to facilitate the use of ratio approach to calculate the utility degree. In contrast, this paper extended and improved the ARAS method in the NTSF environment. This method used the NTSFAAWHM operator to replace the simple weighted summation approach. The reason is that the proposed operator can capture the interrelationship between attributes. Moreover, the NTSF Hamming distance was applied to obtain the utility degree of each alternative. It can not only achieve defuzzification, but also get the utility degree. A comparison of the features of these methods is presented in Table 16. Through the above comparative analysis, the proposed NTSF ARAS-H method has strong advantages in decision-making process and results.

Table 16
Comparison of features with existing ARAS methods

Features		Mishra *et al.* (2020)[44]	Gül (2021a)[45]	Mishra and Rani(2021)[56]	Jovčić *et al.*[43](2020)	Gül (2021b)[46]	This paper
Decision-making environments		IFSs	FFSs	q-ROFSs	PFSs	SFSs	NTSFSs
Information description	Uncertain	Yes	Yes	Yes	Yes	Yes	Yes
	Vague	Yes	Yes	Yes	Yes	Yes	Yes
	Random	No	No	No	No	No	Yes
Defuzzification techniques		Score function	Score function	Score function	Score function	Two-stepdefizzifi- cation method	Hamming distance
Information operation laws		Algebraic	Algebraic	Algebraic	Algebraic	Algebraic	Aczel-Alsina
Whether it considers interrelationship of any attributes?		No	No	No	No	No	Yes
Decision-making flexible		No	No	Weak	No	No	Strong

6. Conclusions

In this article, we defined the Hamming distance measure of NTSFNs. The AA operational rules of NTSFNs were proposed via combining the advantages of NTSFNs and AA TT. Based on this, the NTSFAAHM and NTSFAAWHM operators were developed, and their related properties and special cases were discussed. For the NTSF MAGDM problems, then, we defined similarity, constructed MDM and extended SWARA approach based on the NTSF Hamming distance under NTSF environment, and used them to determine expert weight with regard to attributes, and attribute subjective and objective weight respectively. Further, we improved the ARAS method with the NTSFAAWHM operator and NTSF Hamming distance under NTSF environment, which has the ability to capture the interrelationship between attributes. Finally, the proposed NTSF ARAS-H method was applied to the real case of internet platform investment for waste clothing recycling. The reliability and effectiveness of the proposed method were tested via sensitivity and comparative analysis. Therefore, the main advantages are as follows:

(1) Compared with the existing NTSF distance [11], the NTSF Hamming distance we defined contains the refusal degree of NTSFN, which makes the measurement result more reasonable. Based on this, we defined the similarity, constructed the MDM and improved the SWARA method.

(2) The NTSFAAHM and NTSFAAWHM operators were developed based on the AA operational laws of NTSFNs, which can capture the interrelationship between attributes compared with the NTSFWA and NTSFWG operators and have more adjustable parameters than the NTSFWMSM operator.

(3) Compared with the existing ARAS methods, the NTSF ARAS-H method combined the NTSFAAWHM operator can not only consider the correlation between attributes, but also make the decision process more flexible and reasonable.

(4) In the NTSF ARAS-H method, we used Hamming distance to calculate the distance between each alternative and the ideal solution as the utility degree, which can avoid the loss of some information caused by the defuzzification of the optimal function of each alternative. The ARAS-H method is more comprehensive and reasonable.

The ARAS-H method can be well applied to the MAGDM problems in which attribute variables are represented by NTSFNs. However, there are still some shortcomings in this method. The NTSFAAWHM operator may be computationally complex and difficult when aggregating a large number of input arguments. In addition, this operator can only capture two input arguments correlations, but not more than two input arguments interrelationships. Therefore, we will consider the input arguments hierarchy in the NTSF environment and use the proposed NTSFAAWHM operator for aggregation in the future. Furthermore, on the basis of NTSF AA operations, we develop aggregation operators that can capture interrelationships of multiple aggregated arguments, such as Hamy mean [71], Muirhead mean [31], Maclaurim Symmetric mean [32], etc. At the same time, we will combine the prospect theory or regret theory with WASPAS [72], EDAS [73], CoCoSo[74] and MARCOS [75], and then apply them to different decision-making scenarios to solve practical problems, such as business decision-making, capital account selection and emergency decision-making, etc.

Author Contributions

Conceptualization, H.W.; methodology, H.W.; validation, W.Z.; formal analysis, W.Z.; investigation, H.W.; resources, H.W.; writing—original draft preparation, H.W. and W.Z.; writing—review and editing, H.W. and W.Z.; visualization, W.Z.; supervision, H.W.; project administration, H.W. All authors have read and agreed to the published version of the manuscript.

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Data Availability Statement

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

Conflicts of Interest
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

Proof: Based on the AA operational rules of NTSFNs in Definition 9, we have

$$(\delta_i)^s = \left(\begin{array}{l} (\alpha_i^s, s^{\frac{1}{2}}\alpha_i^{s-1}\sigma_i), \\ \left(\sqrt[q]{\exp\left\{-\left(s\left(-\ln(\tau_i^q)\right)^{\varphi}\right)^{1/\varphi}\right\}}, \sqrt[q]{1-\exp\left\{-\left(s\left(-\ln(1-\eta_i^q)\right)^{\varphi}\right)^{1/\varphi}\right\}}, \sqrt[q]{1-\exp\left\{-\left(s\left(-\ln(1-\vartheta_i^q)\right)^{\varphi}\right)^{1/\varphi}\right\}} \right) \end{array} \right)$$

$$(\delta_j)^t = \left(\begin{array}{l} (\alpha_j^t, t^{\frac{1}{2}}\alpha_j^{t-1}\sigma_j), \\ \left(\sqrt[q]{\exp\left\{-\left(t\left(-\ln(\tau_j^q)\right)^{\varphi}\right)^{1/\varphi}\right\}}, \sqrt[q]{1-\exp\left\{-\left(t\left(-\ln(1-\eta_j^q)\right)^{\varphi}\right)^{1/\varphi}\right\}}, \sqrt[q]{1-\exp\left\{-\left(t\left(-\ln(1-\vartheta_j^q)\right)^{\varphi}\right)^{1/\varphi}\right\}} \right) \end{array} \right)$$

And

$$(\delta_i)^s \otimes_{AA} (\delta_j)^t = \left(\begin{array}{l} \left(\alpha_i^s\alpha_j^t, \alpha_i^s\alpha_j^t\sqrt{\frac{s\sigma_i^2}{\alpha_i^2} + \frac{t\sigma_j^2}{\alpha_j^2}} \right), \\ \left(\begin{array}{l} \sqrt[q]{\exp\left\{-\left[s\left(-\ln(\tau_i^q)\right)^{\varphi} + t\left(-\ln(\tau_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}}, \sqrt[q]{1-\exp\left\{-\left[s\left(-\ln(1-\eta_i^q)\right)^{\varphi} + t\left(-\ln(1-\eta_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}}, \\ \sqrt[q]{1-\exp\left\{-\left[s\left(-\ln(1-\vartheta_i^q)\right)^{\varphi} + t\left(-\ln(1-\vartheta_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}} \end{array} \right) \end{array} \right)$$

Then, we have

$$\oplus_{\substack{AA \\ j=1}}^{n}\left((\delta_i)^s \otimes_{AA} (\delta_j)^t\right)=\left(\begin{array}{l}\left(\sum_{j=1}^{n}\alpha_i^s\alpha_j^t,\sum_{j=1}^{n}\alpha_i^s\alpha_j^t\sqrt{\frac{s\sigma_i^2}{\alpha_i^2}+\frac{t\sigma_j^2}{\alpha_j^2}}\right),\\ \left(\begin{array}{l}\sqrt[q]{1-\exp\left\{-\left[\sum_{j=1}^{n}\left(-\ln\left(1-\exp\left\{-\left[s\left(-\ln(\tau_i^q)\right)^{\varphi}+t\left(-\ln(\tau_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}},\\ \sqrt[q]{\exp\left\{-\left[\sum_{j=1}^{n}\left(-\ln\left(1-\exp\left\{-\left[s\left(-\ln(1-\eta_i^q)\right)^{\varphi}+t\left(-\ln(1-\eta_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}},\\ \sqrt[q]{\exp\left\{-\left[\sum_{j=1}^{n}\left(-\ln\left(1-\exp\left\{-\left[s\left(-\ln(1-\eta_i^q)\right)^{\varphi}+t\left(-\ln(1-\eta_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}}\end{array}\right)\end{array}\right)$$

And

$$\oplus_{\substack{AA \\ i=1,i=j}}^{n}\left((\delta_i)^s \otimes_{AA} (\delta_j)^t\right)=\left(\begin{array}{l}\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i^s\alpha_j^t,\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i^s\alpha_j^t\sqrt{\frac{s\sigma_i^2}{\alpha_i^2}+\frac{t\sigma_j^2}{\alpha_j^2}}\right),\\ \left(\begin{array}{l}\sqrt[q]{1-\exp\left\{-\left[\sum_{\substack{i=1,\\ j=i}}^{n}\left(-\ln\left(1-\exp\left\{-\left[s\left(-\ln(\tau_i^q)\right)^{\varphi}+t\left(-\ln(\tau_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}},\\ \sqrt[q]{\exp\left\{-\left[\sum_{\substack{i=1,\\ j=i}}^{n}\left(-\ln\left(1-\exp\left\{-\left[s\left(-\ln(1-\eta_i^q)\right)^{\varphi}+t\left(-\ln(1-\eta_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}},\\ \sqrt[q]{\exp\left\{-\left[\sum_{\substack{i=1,\\ j=i}}^{n}\left(-\ln\left(1-\exp\left\{-\left[s\left(-\ln(1-\eta_i^q)\right)^{\varphi}+t\left(-\ln(1-\eta_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}}\end{array}\right)\end{array}\right)$$

Further,

$$\frac{2}{n(n+1)}\oplus_{AA\, i=1,j=i}^{n}\left((\delta_i)^s\otimes_{AA}(\delta_j)^t\right)=\left(\begin{array}{l}\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i^s\alpha_j^t,\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i^s\alpha_j^t\sqrt{\frac{s\sigma_i^2}{\alpha_i^2}+\frac{t\sigma_j^2}{\alpha_j^2}}\right),\\ \left(\begin{array}{l}\sqrt[q]{1-\exp\left\{-\left[\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[s\left(-\ln(\tau_i^q)\right)^{\varphi}+t\left(-\ln(\tau_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}},\\ \sqrt[q]{\exp\left\{-\left[\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[s\left(-\ln(1-\eta_i^q)\right)^{\varphi}+t\left(-\ln(1-\eta_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}},\\ \sqrt[q]{\exp\left\{-\left[\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[s\left(-\ln(1-\vartheta_i^q)\right)^{\varphi}+t\left(-\ln(1-\vartheta_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}}\end{array}\right)\end{array}\right)$$

$$\left(\frac{2}{n(n+1)}\oplus_{AA\, i=1,j=i}^{n}\left((\delta_i)^s\otimes_{AA}(\delta_j)^t\right)\right)^{\frac{1}{s+t}}=$$

$$\left(\begin{array}{l}\left(\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i^s\alpha_j^t\right)^{\frac{1}{s+t}},\sqrt{\frac{1}{s+t}}\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i^s\alpha_j^t\right)^{\frac{1}{s+t}-1}\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i^s\alpha_j^t\sqrt{\frac{s\sigma_i^2}{\alpha_i^2}+\frac{t\sigma_j^2}{\alpha_j^2}}\right)\right),\\ \left(\begin{array}{l}\sqrt[q]{\exp\left\{-\left[\frac{1}{s+t}\left(-\ln\left(1-\exp\left\{-\left[\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[s\left(-\ln(\tau_i^q)\right)^{\varphi}+t\left(-\ln(\tau_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}},\\ \sqrt[q]{1-\exp\left\{-\left[\frac{1}{s+t}\left(-\ln\left(1-\exp\left\{-\left[\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[s\left(-\ln(1-\eta_i^q)\right)^{\varphi}+t\left(-\ln(1-\eta_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}},\\ \sqrt[q]{1-\exp\left\{-\left[\frac{1}{s+t}\left(-\ln\left(1-\exp\left\{-\left[\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[s\left(-\ln(1-\eta_i^q)\right)^{\varphi}+t\left(-\ln(1-\eta_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}}\end{array}\right)\end{array}\right)$$

i.e.,

$$NTSFAAHM^{s,t}(\delta_1,\delta_2,\ldots,\delta_n)=\left(\begin{array}{l}\left(\begin{array}{l}\left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\alpha_i^s\alpha_j^t\right)^{\frac{1}{s+t}},\\ \sqrt{\frac{1}{s+t}}\left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\alpha_i^s\alpha_j^t\right)^{\frac{1}{s+t}-1}\left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\alpha_i^s\alpha_j^t\sqrt{\frac{s\sigma_i^2}{\alpha_i^2}+\frac{t\sigma_j^2}{\alpha_j^2}}\right)\end{array}\right),\\ \left(\begin{array}{l}\sqrt[q]{\exp\left\{-\left[\frac{1}{s+t}\left(-\ln\left(1-\exp\left\{-\left[\frac{2T_{ij}}{n(n+1)}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}},\\ \sqrt[q]{1-\exp\left\{-\left[\frac{1}{s+t}\left(-\ln\left(1-\exp\left\{-\left[\frac{2N_{ij}}{n(n+1)}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}},\\ \sqrt[q]{1-\exp\left\{-\left[\frac{1}{s+t}\left(-\ln\left(1-\exp\left\{-\left[\frac{2V_{ij}}{n(n+1)}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}}\end{array}\right)\end{array}\right)$$

where

$$T_{ij}=\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[\left(-\ln(\tau_i^q)\right)^{\varphi}+\left(-\ln(\tau_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi},$$

$$N_{ij}=\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[\left(-\ln(1-\eta_i^q)\right)^{\varphi}+\left(-\ln(1-\eta_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi},$$

$$V_{ij}=\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[\left(-\ln(1-\eta_i^q)\right)^{\varphi}+\left(-\ln(1-\eta_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}.$$

Thus, the proof of Theorem 4 is complete.

Appendix B

(1) (Idempotency)

Proof: Since $\delta_i = \delta$ for all i, we have

$$NTSFAAHM^{s,t}(\delta_1,\delta_2,\ldots,\delta_n)=\left(\frac{2}{n(n+1)}\oplus_{AA}{}_{i=1,j=i}^{n}\delta_i^s\otimes_{AA}\delta_j^t\right)^{\frac{1}{s+t}}$$

$$=\left(\frac{2}{n(n+1)}\oplus_{AA}{}_{i=1,j=i}^{n}\delta^s\otimes_{AA}\delta^t\right)^{\frac{1}{s+t}}$$

$$=\left(\frac{2}{n(n+1)}\oplus_{AA}{}_{i=1,j=i}^{n}\delta^{s+t}\right)^{\frac{1}{s+t}}=\delta$$

Thus, $NTSFAAHM^{s,t}(\delta_1,\delta_2,\ldots,\delta_n)=\delta$.

(2) (Boundedness)

Proof: Since $P^-=\min\{\delta_i\}$, according to the idempotency property of NTSFAAHM, we can get

$$NTSFAAHM^{s,t}(\delta_1,\delta_2,\ldots,\delta_n)=\left(\frac{2}{n(n+1)}\oplus_{AA\,i=1,j=i}^{n}\delta_i^s\otimes_{AA}\delta_j^t\right)^{\frac{1}{s+t}}$$
$$\geq\left(\frac{2}{n(n+1)}\oplus_{AA\,i=1,j=i}^{n}(P^-)^s\otimes_{AA}(P^-)^t\right)^{\frac{1}{s+t}}$$
$$=\left(\frac{2}{n(n+1)}\oplus_{AA\,i=1,j=i}^{n}(P^-)^{s+t}\right)^{\frac{1}{s+t}}=P^-$$

Similarly, we have $NTSFAAHM^{s,t}(\delta_1,\delta_2,\ldots,\delta_n)\leq P^+$.

So, $P^-\leq NTSFAAHM^{s,t}(\delta_1,\delta_2,\ldots,\delta_n)\leq P^+$.

(3) (Monotonicity)

Proof: Since $\alpha_i\leq\alpha_i^*$, $\tau_i\leq\tau_i^*$, $\eta_i\geq\eta_i^*$ and $\vartheta_i\geq\vartheta_i^*$ for any i, then we can get

$$\left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\alpha_i^s\alpha_j^t\right)^{\frac{1}{s+t}}\leq\left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}(\alpha_i^*)^s(\alpha_j^*)^t\right)^{\frac{1}{s+t}};$$

$$\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[s\left(-\ln(\tau_i^q)\right)^{\varphi}+t\left(-\ln(\tau_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\leq\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[s\left(-\ln((\tau_i^*)^q)\right)^{\varphi}+t\left(-\ln((\tau_j^*)^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi};$$

$$\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[s\left(-\ln(1-\eta_i^q)\right)^{\varphi}+t\left(-\ln(1-\eta_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\geq$$
$$\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[s\left(-\ln(1-(\eta_i^*)^q)\right)^{\varphi}+t\left(-\ln(1-(\eta_j^*)^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi};$$

$$\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[s\left(-\ln(1-\vartheta_i^q)\right)^{\varphi}+t\left(-\ln(1-\vartheta_j^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\geq$$
$$\sum_{i=1,j=i}^{n}\left(-\ln\left(1-\exp\left\{-\left[s\left(-\ln(1-(\vartheta_i^*)^q)\right)^{\varphi}+t\left(-\ln(1-(\vartheta_j^*)^q)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}.$$

Then,

$$\sqrt[q]{\exp\left\{-\left[\frac{1}{s+t}\left(-\ln\left(1-\exp\left\{-\left[\frac{2T_{ij}}{n(n+1)}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}}\leq\sqrt[q]{\exp\left\{-\left[\frac{1}{s+t}\left(-\ln\left(1-\exp\left\{-\left[\frac{2T_{ij}^*}{n(n+1)}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}}$$

;

$$\sqrt[q]{1-\exp\left\{-\left[\frac{1}{s+t}\left(-\ln\left(1-\exp\left\{-\left[\frac{2N_{ij}}{n(n+1)}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}}\geq\sqrt[q]{1-\exp\left\{-\left[\frac{1}{s+t}\left(-\ln\left(1-\exp\left\{-\left[\frac{2N_{ij}^*}{n(n+1)}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}};$$

$$\sqrt[q]{1-\exp\left\{-\left[\frac{1}{s+t}\left(-\ln\left(1-\exp\left\{-\left[\frac{2V_{ij}}{n(n+1)}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}} \geq \sqrt[q]{1-\exp\left\{-\left[\frac{1}{s+t}\left(-\ln\left(1-\exp\left\{-\left[\frac{2V_{ij}^{*}}{n(n+1)}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right]^{1/\varphi}\right\}}.$$

According to the score function in definition 3, we obtain $NTSFAAHM^{s,t}(\delta_1,\delta_2,\ldots,\delta_n)\leq NTSFAAHM^{s,t}(\delta_1^*,\delta_2^*,\ldots,\delta_n^*)$.

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