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An Integrated Fermatean Fuzzy Group Decision Method based on Sugeno-Weber Operators for the Selection of Reverse Logistics Supplier

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ABSTRACT

The implementation of reverse logistics not only enables enterprises to reduce costs and mitigate operational risks but also enhances their core competitiveness. However, selecting the optimal reverse logistics supplier remains a critical decision for enterprises and a pressing challenge in modern enterprise and logistics management. To address this issue, this paper introduces a novel multiple criteria group decision-making (MCGDM) approach that integrates the coefficient of variation method, the weighted aggregated sum product assessment (WASPAS) method, and newly proposed aggregation operators within the Fermatean fuzzy framework. Firstly, we define Sugeno-Weber operations for Fermatean fuzzy numbers (FFNs) and subsequently propose four novel aggregation operators: the Fermatean fuzzy Sugeno-Weber weighted averaging operator, the Fermatean fuzzy Sugeno-Weber weighted geometric operator, and their respective ordered weighted variants. These operators are then employed to aggregate the Fermatean fuzzy assessment information provided by decision-making experts. Next, the coefficient of variation method is introduced, utilizing the Fermatean fuzzy score function to determine the importance weights of the assessment criteria. Finally, an enhanced WASPAS method is proposed to rank the alternatives. To demonstrate the effectiveness and feasibility of the proposed group decision-making method, a case study on green supplier selection is presented and analyzed. This study not only provides a robust framework for supplier selection but also contributes to the advancement of decision-making methodologies in the context of reverse logistics.

1. Introduction

With the rapid development of the circular economy, reverse logistics has gained significant attention from enterprises due to its unique advantages in transportation efficiency and resource utilization. An increasing number of companies are enhancing their resource reuse capabilities and strengthening their core competitiveness by improving their reverse logistics management systems.

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As a critical foundation for enterprise growth and the achievement of high-quality socio-economic development, reverse logistics transportation plays a pivotal role in modern business operations. To reduce costs and mitigate operational risks, most enterprises opt to outsource their reverse logistics operations to specialized suppliers. Effective reverse logistics management not only helps reduce environmental pollution but also lowers operational costs, thereby gradually enhancing a company's competitive advantage[1]. Consequently, selecting the optimal reverse logistics supplier has become a crucial decision for enterprises and a pressing challenge in modern enterprise and logistics management. When evaluating reverse logistics suppliers, it is essential to consider a wide range of quantitative and qualitative factors. Additionally, multiple decision-making experts must collaborate to avoid the subjectivity inherent in individual judgments. As such, the selection of reverse logistics suppliers can be characterized as a typical multiple criteria decision-making (MCDM) problem [2]. Furthermore, given the inherent uncertainty in experts' preferences and judgments, it is necessary to establish an uncertain multi-criteria group decision-making (MCGDM) model integrated with a fuzzy information representation framework. This approach ensures the identification of suitable reverse logistics suppliers in a robust and systematic manner.

The essence of MCGDM is the decision-making process in which a group of experts, based on their cognitive abilities and given evaluation criteria, select and prioritize alternative solutions. Due to the differences in knowledge level and experience among expert groups, it is difficult to evaluate the information of reverse logistics suppliers under different conflict criteria, and there is a certain degree of ambiguity and uncertainty. In order to more reasonably assist decision-making experts in providing their preference information, Zadeh[2] introduced the concept of fuzzy set (FSs), which provides an effective solution for people to deal with fuzzy information in the decision-making process and provides new solutions for practical problems such as fuzzy control[3, 4, 5], medical diagnosis[6, 7, 8], and decision analysis[9, 10, 11, 12]. However, with the increasing complexity of decision-making problems, researchers have found that traditional fuzzy set theory is no longer applicable to solving all MCDM problems. Subsequently, research based on FS theory has received widespread attention from scholars and has successively proposed intuitionistic FS [13], interval-valued intuitionistic FS [14], type-2 FS [15], and Pythagorean FS [16]. Intuitionistic FS, as an effective extension of classical FS, enhance their ability to characterize uncertainty by considering the membership and non-membership of elements to a given target. In order to handle all fuzzy information, Yager[17] proposed q-rung orthogonal FS in 2016 to expand the space for decision makers to provide preference information. As a generalized form of intuitionistic FS, q-rung orthogonal FS characterizes and processes all fuzzy information based on the property that the sum of its membership degree to the power q and non-membership degree to the power q is not greater than 1. Afterwards, Senapati and Yager[18] proposed Fermatean FS, which have a constraint on membership and non-membership that the sum of membership to the third power and non-membership to the third power is not greater than 1. Compared to intuitionistic FS and Pythagorean FS, it has a wider range of information representation. Since the proposal of FFS, scholars have continued the research approach of intuitionistic FS and obtained rich research results, involving information measurement [19, 20, 21], aggregation operator[22, 23, 24, 25], fuzzy set extension[26, 27], and decision analysis[28, 29, 30, 31], applications [32, 33]. Aggregation operator, as an important information fusion theory has been investigated by scholars to construct decision approaches. The fundamental of aggregation operator is to define operational laws using different triangular norms. In the existing research, the Dombi norms, Einstein norms, Hamacher norms, Frank norms has been used for propounding aggregation operators. Recently, the Sugeno-Weber triangular is extended to t-spherical fuzzy hypersoft environment for proposing some novel powerful aggregation operators[34]. Further, Senapati et al.[35] proposed some dual hesitant q-rung orthopair fuzzy Sugeno-Weber operators and constructed decision approach for enhancing healthcare supply chain management. In the current research of FFS, there is no work on Sugeno-Weber operator and its applications.

Zavadskas et al.[36] introduced the WASPAS (Weighted Aggregated Sum Product Assessment) method, which integrates weighted sum models and weighted product models to address decision-making problems. They demonstrated the advantages of the WASPAS method over several existing approaches, establishing it as a unique and user-friendly decision-making tool. Since its introduction, the WASPAS method has been widely applied across various fields. Owing to its effectiveness in decision analysis, it has been extended to solve diverse evaluation and decision-making problems in multiple uncertain environments[37, 38, 39, 40, 41]. Mardani et al. [42] provided a comprehensive review and analysis of the SWARA (Step-wise Weight Assessment Ratio Analysis) and WASPAS methods, examining their theoretical foundations, applications, and developmental trends. Ayvaz et al. [43] developed a novel occupational health and safety risk assessment model by integrating the AHP (Analytic Hierarchy Process)-WASPAS approach with the Fine-Kinney method, specifically for evaluating occupational hazards in the aquaculture sector. These studies collectively underscore the robustness, feasibility, and operational simplicity of the WASPAS method in practical decision-making scenarios. However, despite its widespread adoption, existing extensions of the WASPAS method have not yet incorporated Sugeno-Weber operators within the Fermatean fuzzy framework. Recent advancements have explored alternative extensions and improvements. For instance, Görçün et al. [44] proposed an enhanced WASPAS method based on the Bonferroni mean under a Type-2 neutrosophic setting, applied to the selection of appropriate Ro-Ro vessels in the second-hand market. Rong et al.[45] introduced an integrated group decision-making framework combining MULTIMOORA (Multi-Objective Optimization by Ratio Analysis) and WASPAS under q-rung orthopair fuzzy information for evaluating third-party reverse logistics providers. Additionally, Akram et al.[46] presented a 2-tuple linguistic Fermatean fuzzy MAGDM (Multi-Attribute Group Decision-Making) approach based on the WASPAS method to select optimal solid waste disposal locations. These developments highlight the versatility and adaptability of the WASPAS method, while also indicating opportunities for further innovation, particularly through the integration of Sugeno-Weber operators in Fermatean fuzzy environments.

The primary objective of this article is to establish a multi-criteria group decision-making (MCGDM) method in an uncertain environment and develop an evaluation model for reverse logistics supplier selection, thereby assisting enterprises in making informed and rational decisions. Based on a comprehensive review of Fermatean fuzzy sets (FFS) and decision-making approaches, the contributions of this paper are outlined as follows:

(1) We define the Sugeno-Weber operations for Fermatean fuzzy numbers (FFNs) and propose four novel aggregation operators: the Fermatean fuzzy Sugeno-Weber weighted averaging operator, the Fermatean fuzzy Sugeno-Weber weighted geometric operator, and their corresponding ordered weighted variants.

(2) A Fermatean fuzzy coefficient of variation method is introduced, leveraging the proposed aggregation operators and distance measures, to determine the weights of evaluation criteria.

(3) Building on the proposed Fermatean fuzzy Sugeno-Weber aggregation operators, an enhanced version of the classical WASPAS (Weighted Aggregated Sum Product Assessment) method is presented for ranking alternatives.

(4) A case study on green supplier selection is conducted to demonstrate the effectiveness and feasibility of the proposed group decision-making method.

The structure of this research is organized as follows: Section 2 introduces the fundamental concepts of Fermatean fuzzy sets (FFS) and the definition of Sugeno-Weber norms. Section 3 presents novel Sugeno-Weber operators under the Fermatean fuzzy environment. Section 4 proposes a new MAGDM approach and elaborates on its implementation process. Section 5 conducts a case study to validate the effectiveness of the proposed methodology. Finally, Section 6 summarizes the key conclusions of this study.

2. Prerequisites

This section introduces conceptions of Fermatean fuzzy set including definition, comparison laws and aggregation operators.

Definition 1[18]. Let U be a fix set. Then, an FFS Ξ on U is stated as below:

$$\Xi = \{< u, \xi_{\Xi}(u), \kappa_{\Xi}(u) > | u \in U\}, \tag{1}$$

in which the function $\xi_{\Xi}(u): U \to [0,1], \kappa_{\Xi}(u): U \to [0,1]$ with the following condition $0 \le (\xi_{\Xi}(u))^3 + (\kappa_{\Xi}(u))^3 \le 1$, for $u \in U$. The function $\xi_{\Xi}(u), \kappa_{\Xi}(u)$ severally signify the approval grade and disapproval grade of element u in Ξ. For simplicity, the pair $\Xi = (\xi_{\Xi}, \kappa_{\Xi})$ is call Fermatean fuzzy number (FFN), which holds $\xi_{\Xi}, \kappa_{\Xi} \in [0,1]$ and $0 \le (\xi_{\Xi})^2 + (\kappa_{\Xi})^2 \le 1$. The hesitant grade is depicted as $\pi_{\Xi}(u) = \sqrt[3]{1 - (\xi_{\Xi}(u))^3 - (\kappa_{\Xi}(u))^3}$.

Definition 2[18]. Given two FFNs $\Xi_1 = (\xi_1, \kappa_1)$ and $\Xi_2 = (\xi_2, \kappa_2)$. Then the Fermatean fuzzy interaction operational laws are defined as

(1) $\Xi_1 \oplus \Xi_2 = \left(\sqrt[3]{(\xi_1)^3 + (\xi_2)^3 - (\xi_1)^3 (\xi_2)^3}, \kappa_1 \kappa_2 \right);$

(2) $\Xi_1 \otimes \Xi_2 = \left(\xi_1 \xi_2, \sqrt[3]{(\kappa_1)^3 + (\kappa_2)^3 - (\kappa_1)^3 (\kappa_2)^3} \right);$

(3) $\lambda \Xi_1 = \left(\sqrt[3]{1 - \left(1 - (\xi_1)^3\right)^{\lambda}}, (\kappa_1)^{\lambda} \right), \lambda > 0;$

(4) $\lambda \Xi_1 = \left((\xi_1)^{\lambda}, \sqrt[3]{1 - \left(1 - (\kappa_1)^3\right)^{\lambda}} \right), \lambda > 0.$

Definition 3[22]. Let $\Xi = (\xi, \kappa)$ be an FFN. The score and accuracy index of F are stated as

$$SI(\Xi) = \frac{1}{2}(\phi^3 - \kappa^3 + 1), SI(\Xi) \in [0,1] , \tag{2}$$

$$AI(\Xi) = \xi^3 + \kappa^3, AI(\Xi) \in [0,1]. \tag{3}$$

Based on the score and accuracy index of FFN, the comparison rules are defined as follows.

Definition 4[22]. Given two FFNs $\Xi_1 = (\xi_1, \kappa_1)$ and $\Xi_2 = (\xi_2, \kappa_2)$. To compare these FFNs, we have

If $SI(\Xi_1) > SI(\Xi_2)$, then $\Xi_1 \succ \Xi_2$,

If $SI(\Xi_1) = SI(\Xi_2)$, then

If $AI(\Xi_1) < AI(\Xi_2)$, then $\Xi_1 \prec \Xi_2$;

If $AI(\Xi_1) = AI(\Xi_2)$, then $\Xi_1 \sim \Xi_2$.

Definition 5[22]. Suppose that $\Xi_j = (\xi_j, \kappa_j)(j = 1(1)n)$ be a group of FFNs. Then the Fermatean fuzzy weighted averaging (FFWA) and geometric (FFWG) operators are depicted by

$$FFWA(\Xi_1, \Xi_2, \cdots, \Xi_n) = \bigoplus_{j=1}^{n} (\varpi_j \Xi_j) = \left(\sqrt[3]{1 - \prod_{j=1}^{n} \left(1 - (\xi_j)^3\right)^{\varpi_j}}, \prod_{j=1}^{n} (\kappa_j)^{\varpi_j} \right), \tag{4}$$

$$FFWG(\Xi_1, \Xi_2, \cdots, \Xi_n) = \bigotimes_{j=1}^{n} (\Xi_j)^{\varpi_j} = \left(\prod_{j=1}^{n} (\xi_j)^{\varpi_j}, \sqrt[3]{1 - \prod_{j=1}^{n} \left(1 - (\kappa_j)^3\right)^{\varpi_j}} \right). \tag{5}$$

where ϖ_j is the weight of Ξ_j with $\sum_{j=1}^{n} \varpi_j = 1,\ \varpi_j \in [0, 1]$.

Definition 6[47]. The Sugeno-Weber t-norm $(T_{SW}^{\Re})_{\Re \in [0,\infty)}$ and t-conorm $(S_{SW}^{\Re})_{\Re \in [0,\infty)}$ are defined as follows:

$$T_{SW}^{\Re}(a,b) = \begin{cases} T_D(a,b), \text{if } \Re = -1 \\ \max\left(0, \dfrac{a+b-1+\Re ab}{1+\Re}\right), \text{if } -1 < \Re < +\infty, \\ T_P(a,b), \text{if } \Re = +\infty \end{cases} \tag{6}$$

wherein $T_D(a,b)$ and $T_P(a,b)$ denote the drastic t-norm and product t-norm, severally.

$$S_{SW}^{\Re} = \begin{cases} S_D(a,b), \text{if } \Re = -1 \\ \min\left(1, a+b-\dfrac{\Re}{1+\Re}ab\right), \text{if } -1 < \Re < +\infty \\ S_P(a,b), \text{if } \Re = +\infty \end{cases} \tag{7}$$

wherein $S_D(a,b)$ and $S_P(a,b)$ denote the drastic t-norm and probabilistic sum, severally.

3. Some novel Fermatean fuzzy Sugeno-Weber aggregation operators

This section first proposed the Fermatean fuzzy Sugeno-Weber operations based on Sugeno-Weber norms. Then four weighted averaging and geometric operators are advanced for aggregating Fermatean fuzzy information. The related properties of the proposed operators are also explored.

3.1 Sugeno-Weber Operations on Fermatean fuzzy numbers

Definition 7. Given two FFNs $\Xi_1=(\xi_1,\kappa_1)$ and $\Xi_2=(\xi_2,\kappa_2)$. Then the Fermatean fuzzy Sugeno-Weber operational laws are defined as

(1) $\Xi_1\oplus_{SW}\Xi_2=\left(\sqrt[3]{(\xi_1)^3+(\xi_2)^3-\frac{\Re}{1+\Re}(\xi_1)^3(\xi_2)^3},\sqrt[3]{\frac{(\kappa_1)^3+(\kappa_2)^3-1+\Re(\kappa_1)^3(\kappa_2)^3}{1+\Re}}\right);$

(2) $\Xi_1\otimes_{SW}\Xi_2=\left(\sqrt[3]{\frac{(\xi_1)^3+(\xi_2)^3-1+\Re(\xi_1)^3(\xi_2)^3}{1+\Re}},\sqrt[3]{(\kappa_1)^3+(\kappa_2)^3-\frac{\Re}{1+\Re}(\kappa_1)^3(\kappa_2)^3},\right);$

(3) $\lambda\Xi_1=\left(\sqrt[3]{\frac{1+\Re}{\Re}\left(1-\left(1-(\xi_1)^3\left(\frac{\Re}{1+\Re}\right)\right)^{\lambda}\right)},\sqrt[3]{\frac{1}{\Re}\left((1+\Re)\left(\frac{\Re(\kappa_1)^3+1}{1+\Re}\right)^{\lambda}-1\right)}\right),\lambda>0;$

(4) $(\Xi_1)^{\lambda}=\left(\sqrt[3]{\frac{1}{\Re}\left((1+\Re)\left(\frac{\Re(\xi_1)^3+1}{1+\Re}\right)^{\lambda}-1\right)},\sqrt[3]{\frac{1+\Re}{\Re}\left(1-\left(1-(\kappa_1)^3\left(\frac{\Re}{1+\Re}\right)\right)^{\lambda}\right)}\right),\lambda>0.$

Theorem 1. Let $\Xi_1=(\xi_1,\kappa_1)$ and $\Xi_2=(\xi_2,\kappa_2)$ be two FFNs and $\lambda,\lambda_1,\lambda_2>0$. then the following characteristics can be attained:

(1) $\Xi_1\oplus\Xi_2=\Xi_2\oplus\Xi_1;$

(2) $\Xi_1\otimes\Xi_2=\Xi_2\otimes\Xi_1;$

(3) $\lambda(\Xi_1\oplus\Xi_2)=\lambda\Xi_1\oplus\lambda\Xi_2;$

(4) $(\Xi_1\otimes\Xi_2)^{\lambda}=(\Xi_1)^{\lambda}\otimes(\Xi_2)^{\lambda};$

(5) $(\lambda_1+\lambda_2)\Xi_1=\lambda_1\Xi_1\oplus\lambda_2\Xi_1;$

(6) $(\Xi_1)^{(\lambda_1+\lambda_2)}=(\Xi_1)^{\lambda_1}\otimes(\Xi_1)^{\lambda_2}.$

Proof. It is trivial.

3.2 Fermatean fuzzy Sugeno-Weber averaging operators

The section proposes the Sugeno-Weber averaging operator on FFN.

Definition 8. Suppose that $\Xi_j=(\xi_j,\kappa_j)(j=1(1)n)$ be a group of FFNs. The mapping $FFSWWA:\Delta^n\to\Delta$ is defined

$$FFSWWA(\Xi_1,\Xi_2,\cdots,\Xi_n)=(\varpi_1\Xi_1)\oplus_{SW}(\varpi_2\Xi_2)\oplus_{SW}\cdots\oplus_{SW}(\varpi_n\Xi_n), \quad (8)$$

where FFSWWA is called as Fermatean fuzzy Sugeno-Weber weighted averaging operator. where ϖ_j is the weight of Ξ_j with $\sum_{j=1}^{n}\varpi_j=1,\ \varpi_j\in[0,1]$.

Theorem 2. The aggregation outcome of the set of FFNs $\Xi_j\left(j=1(1)n\right)$ utilizing FFSWWA operator is an FFN and depicted as

$$FFSWWA\left(\Xi_1,\Xi_2,\cdots,\Xi_n\right)=\left(\sqrt[3]{\frac{1+\Re}{\Re}\left(1-\prod_{j=1}^{n}\left(1-\left(\xi_j\right)^3\left(\frac{\Re}{1+\Re}\right)\right)^{\varpi_j}\right)},\sqrt[3]{\frac{1}{\Re}\left(\left(1+\Re\right)\prod_{j=1}^{n}\left(\frac{\Re\left(\kappa_j\right)^3+1}{1+\Re}\right)^{\varpi_j}-1\right)}\right). \quad (9)$$

Proof. With the help of mathematical induction method, the Theorem 2 can be proved as below:
When $n=2$, we have

$$\varpi_1\Xi_1=\left(\sqrt[3]{\frac{1+\Re}{\Re}\left(1-\left(1-\left(\xi_1\right)^3\left(\frac{\Re}{1+\Re}\right)\right)^{\varpi_1}\right)},\sqrt[3]{\frac{1}{\Re}\left(\left(1+\Re\right)\left(\frac{\Re\left(\kappa_1\right)^3+1}{1+\Re}\right)^{\varpi_1}-1\right)}\right)$$

$$\varpi_2\Xi_2=\left(\sqrt[3]{\frac{1+\Re}{\Re}\left(1-\left(1-\left(\xi_2\right)^3\left(\frac{\Re}{1+\Re}\right)\right)^{\varpi_2}\right)},\sqrt[3]{\frac{1}{\Re}\left(\left(1+\Re\right)\left(\frac{\Re\left(\kappa_2\right)^3+1}{1+\Re}\right)^{\varpi_2}-1\right)}\right)$$

Then

$$FFSWWA\left(\Xi_1,\Xi_2\right)=\left(\varpi_1\Xi_1\right)\oplus_{SW}\left(\varpi_2\Xi_2\right)$$

$$=\left(\begin{array}{l}\sqrt[3]{\frac{1+\Re}{\Re}\left(1-\left(1-\left(\xi_1\right)^3\left(\frac{\Re}{1+\Re}\right)\right)^{\varpi_1}\right)}\oplus_{SW}\sqrt[3]{\frac{1+\Re}{\Re}\left(1-\left(1-\left(\xi_2\right)^3\left(\frac{\Re}{1+\Re}\right)\right)^{\varpi_2}\right)},\\ \sqrt[3]{\frac{1}{\Re}\left(\left(1+\Re\right)\left(\frac{\Re\left(\kappa_1\right)^3+1}{1+\Re}\right)^{\varpi_1}-1\right)}\oplus_{SW}\sqrt[3]{\frac{1}{\Re}\left(\left(1+\Re\right)\left(\frac{\Re\left(\kappa_2\right)^3+1}{1+\Re}\right)^{\varpi_2}-1\right)}\end{array}\right)$$

$$=\left(\sqrt[3]{\frac{1+\Re}{\Re}\left(1-\prod_{j=1}^{2}\left(1-\left(\xi_j\right)^3\left(\frac{\Re}{1+\Re}\right)\right)^{\varpi_j}\right)},\sqrt[3]{\frac{1}{\Re}\left(\left(1+\Re\right)\prod_{j=1}^{2}\left(\frac{\Re\left(\kappa_j\right)^3+1}{1+\Re}\right)^{\varpi_j}-1\right)}\right).$$

It is supposed that the Eq.(9) holds for $n=n$, then

$$FFSWWA\left(\Xi_1,\Xi_2,\cdots,\Xi_n\right)=\left(\varpi_1\Xi_1\right)\oplus_{SW}\left(\varpi_2\Xi_2\right)\oplus_{SW}\cdots\oplus_{SW}\left(\varpi_n\Xi_n\right)$$

$$=\left(\sqrt[3]{\frac{1+\Re}{\Re}\left(1-\prod_{j=1}^{n}\left(1-\left(\xi_j\right)^3\left(\frac{\Re}{1+\Re}\right)\right)^{\varpi_j}\right)},\sqrt[3]{\frac{1}{\Re}\left(\left(1+\Re\right)\prod_{j=1}^{n}\left(\frac{\Re\left(\kappa_j\right)^3+1}{1+\Re}\right)^{\varpi_j}-1\right)}\right).$$

When $n=n+1$, we have

$$FFSWWA\left(\Xi_1,\Xi_2,\cdots,\Xi_{n+1}\right)=\left(\varpi_1\Xi_1\right)\oplus_{SW}\left(\varpi_2\Xi_2\right)\oplus_{SW}\cdots\oplus_{SW}\left(\varpi_n\Xi_n\right)\oplus_{SW}\left(\varpi_{n+1}\Xi_{n+1}\right)$$

$$=\left(\sqrt[3]{\frac{1+\Re}{\Re}\left(1-\prod_{j=1}^{n}\left(1-\left(\xi_j\right)^3\left(\frac{\Re}{1+\Re}\right)\right)^{\varpi_j}\right)},\sqrt[3]{\frac{1}{\Re}\left(\left(1+\Re\right)\prod_{j=1}^{n}\left(\frac{\Re\left(\kappa_j\right)^3+1}{1+\Re}\right)^{\varpi_j}-1\right)}\right)$$

$$\oplus_{SW}\left(\sqrt[3]{\frac{1+\Re}{\Re}\left(1-\left(1-\left(\xi_{n+1}\right)^3\left(\frac{\Re}{1+\Re}\right)\right)^{\varpi_{n+1}}\right)},\sqrt[3]{\frac{1}{\Re}\left(\left(1+\Re\right)\left(\frac{\Re\left(\kappa_{\widetilde{n+1}}\right)^3+1}{1+\Re}\right)^{\varpi_{\widetilde{n+1}}}-1\right)}\right)$$

$$=\left(\sqrt[3]{\frac{1+\Re}{\Re}\left(1-\prod_{j=1}^{\widetilde{n+1}}\left(1-\left(\xi_j\right)^3\left(\frac{\Re}{1+\Re}\right)\right)^{\varpi_j}\right)},\sqrt[3]{\frac{1}{\Re}\left(\left(1+\Re\right)\prod_{j=1}^{\widetilde{n+1}}\left(\frac{\Re\left(\kappa_j\right)^3+1}{1+\Re}\right)^{\varpi_j}-1\right)}\right).$$

Hence, the Eq. (9) holds for $n=n+1$, namely, Eq. (9) holds for all n. Besides, it is easy to prove that the aggregation value is an FFN. Accordingly, the Theorem 2 is proved.

The related important properties of FFSWWA operator are discussed in the following.

Property 1. (Idempotency). Suppose $\Xi_j=\left(\xi_j,\kappa_j\right)\left(j=1(1)n\right)$ denotes a set of FFNs. If $\Xi_j=\Xi=\left(\xi,\kappa\right)$ for all n, then

$$FFSWWA\left(\Xi_1,\Xi_2,\cdots,\Xi_n\right)=\Xi..$$

Proof. Since $\Xi_j=\Xi=\left(\xi,\kappa\right)$ for all n, then

$$FFSWWA\left(\Xi_1,\Xi_2,\cdots,\Xi_n\right)=$$

$$\left(\sqrt[3]{\frac{1+\Re}{\Re}\left(1-\prod_{j=1}^{n}\left(1-\left(\xi_j\right)^3\left(\frac{\Re}{1+\Re}\right)\right)^{\varpi_j}\right)},\sqrt[3]{\frac{1}{\Re}\left(\left(1+\Re\right)\prod_{j=1}^{n}\left(\frac{\Re\left(\kappa_j\right)^3+1}{1+\Re}\right)^{\varpi_j}-1\right)}\right)$$

$$=\left(\sqrt[3]{\frac{1+\Re}{\Re}\left(1-\left(1-\left(\xi\right)^3\left(\frac{\Re}{1+\Re}\right)\right)^{\sum_{j=1}^{n}\varpi_j}\right)},\sqrt[3]{\frac{1}{\Re}\left(\left(1+\Re\right)\left(\frac{\Re\left(\kappa\right)^3+1}{1+\Re}\right)^{\sum_{j=1}^{n}\varpi_j}-1\right)}\right)$$

$$=\left(\sqrt[3]{\frac{1+\Re}{\Re}\left(1-\left(1-\left(\xi\right)^3\left(\frac{\Re}{1+\Re}\right)\right)\right)},\sqrt[3]{\frac{1}{\Re}\left(\left(1+\Re\right)\left(\frac{\Re\left(\kappa\right)^3+1}{1+\Re}\right)-1\right)}\right)$$

$$=\left(\sqrt[3]{\frac{1+\Re}{\Re}\left(\left(\xi\right)^3\left(\frac{\Re}{1+\Re}\right)\right)},\sqrt[3]{\frac{1}{\Re}\left(\Re\left(\kappa\right)^3\right)}\right)=\left(\xi,\kappa\right).$$

Property 2. (Monotonicity). Suppose $\Xi_j=\left(\xi_j,\kappa_j\right)$ and $\overline{\Xi}_j=\left(\overline{\xi}_j,\overline{\kappa}_j\right)\left(j=1(1)n\right)$ denote two sets of FFNs. If $\Xi_j\leq\overline{\Xi}_j$ for all j, then we attain

$$FFSWWA\left(\Xi_1,\Xi_2,\cdots,\Xi_n\right)\leq FFSWWA\left(\overline{\Xi}_1,\overline{\Xi}_2,\cdots,\overline{\Xi}_n\right).$$

Proof. Based on the definition of FFS, the monotonicity of FFSWWA operator can be proved easily, it is omitted here.

Property 3. (Boundedness). Suppose $\Xi_j = (\xi_j, \kappa_j)(j = 1(1)n)$ denotes a set of FFNs. Let $\Xi^- = \min\{\Xi_1, \Xi_2, \cdots, \Xi_n\}$ and $\Xi^+ = \max\{\Xi_1, \Xi_2, \cdots, \Xi_n\}$. Then we attain

$$\Xi^- \le FFSWWA(\Xi_1, \Xi_2, \cdots, \Xi_n) \le \Xi^+.$$

Proof. Based on the Idempotency of FFSWWA operator, we have

$$FFSWWA(\Xi_1, \Xi_2, \cdots, \Xi_n) = FFSWWA(\Xi^+, \Xi^+, \cdots, \Xi^+) = \Xi^+,$$

$$FFSWWA(\Xi_1, \Xi_2, \cdots, \Xi_n) = FFSWWA(\Xi^-, \Xi^-, \cdots, \Xi^-) = \Xi^-.$$

Besides, with the aid of monotonicity of FFSWWA operator, one has

$$FFSWWA(\Xi_1, \Xi_2, \cdots, \Xi_n) \le FFSWWA(\Xi^+, \Xi^+, \cdots, \Xi^+),$$

$$FFSWWA(\Xi_1, \Xi_2, \cdots, \Xi_n) \ge FFSWWA(\Xi^-, \Xi^-, \cdots, \Xi^-).$$

Hence, we have $\Xi^- \le FFSWWA(\Xi_1, \Xi_2, \cdots, \Xi_n) \le \Xi^+$.

Definition 9. Suppose that $\Xi_j = (\xi_j, \kappa_j)(j = 1(1)n)$ be a group of FFNs. ϖ_j is the weight of Ξ_j with $\sum_{j=1}^{n} \varpi_j = 1,\ \varpi_j \in [0, 1]$. The mapping $FFSWOWA : \Delta^n \to \Delta$ is defined

$$FFSWOWA(\Xi_1, \Xi_2, \cdots, \Xi_n) = (\varpi_1 \Xi_{o(1)}) \oplus_{SW} (\varpi_2 \Xi_{o(2)}) \oplus_{SW} \cdots \oplus_{SW} (\varpi_n \Xi_{o(n)}), \tag{10}$$

where FFSWOWA is called as Fermatean fuzzy Sugeno-Weber ordered weighted averaging operator. $(o(1), o(2), \cdots, o(n))$ is a permutation of $(1, 2, \cdots, n)$ such that $\Xi_{o(j-1)} \ge \Xi_{o(j)}$, $\forall j = 2, 3, \cdots, n$.

Theorem 3. The aggregation outcome of the set of FFNs $\Xi_j\ (j = 1(1)n)$ utilizing FFSWOWA operator is an FFN and depicted as

$$FFSWOWA(\Xi_1, \Xi_2, \cdots, \Xi_n) = \left(\sqrt[3]{\frac{1+\Re}{\Re}\left(1 - \prod_{j=1}^{n}\left(1 - \left(\xi_{o(j)}\right)^3 \left(\frac{\Re}{1+\Re}\right)\right)^{\varpi_j}\right)}, \sqrt[3]{\frac{1}{\Re}\left((1+\Re)\prod_{j=1}^{n}\left(\frac{\Re\left(\kappa_{o(j)}\right)^3 + 1}{1+\Re}\right)^{\varpi_j} - 1\right)} \right). \tag{11}$$

The FFSWOWA operator also has the properties of idempotency, monotonicity, and boundedness

3.3 Fermatean fuzzy Sugeno-Weber geometric operators

Definition 10. Suppose that $\Xi_j = (\xi_j, \kappa_j)(j = 1(1)n)$ be a group of FFNs. The mapping $FFSWWG : \Delta^n \to \Delta$ is defined

$$FFSWWG(\Xi_1, \Xi_2, \cdots, \Xi_n) = (\Xi_1)^{\varpi_1} \oplus_{SW} (\Xi_2)^{\varpi_2} \oplus_{SW} \cdots \oplus_{SW} (\Xi_n)^{\varpi_n}, \tag{12}$$

where FFSWWG is called as Fermatean fuzzy Sugeno-Weber weighted geometric operator. where ϖ_j is the weight of Ξ_j with $\sum_{j=1}^{n} \varpi_j = 1,\ \varpi_j \in [0, 1]$.

Theorem 4. The aggregation outcome of the set of FFNs $\Xi_j\ (j = 1(1)n)$ utilizing FFSWWG operator is an FFN and depicted as

$$FFSWWG\left(\Xi_1,\Xi_2,\cdots,\Xi_n\right)=$$
$$\left(\sqrt[3]{\frac{1}{\Re}\left(\left(1+\Re\right)\prod_{j=1}^{n}\left(\frac{\Re\left(\xi_j\right)^3+1}{1+\Re}\right)^{\varpi_j}-1\right)},\sqrt[3]{\frac{1+\Re}{\Re}\left(1-\prod_{j=1}^{n}\left(1-\left(\kappa_j\right)^3\left(\frac{\Re}{1+\Re}\right)\right)^{\varpi_j}\right)}\right). \quad (13)$$

Definition 11. Suppose that $\Xi_j=\left(\xi_j,\kappa_j\right)\left(j=1(1)n\right)$ be a group of FFNs. ϖ_j is the weight of Ξ_j with $\sum_{j=1}^{n}\varpi_j=1,\ \varpi_j\in[0,1]$. The mapping $FFSWOWG:\Delta^n\to\Delta$ is defined

$$FFSWOWG\left(\Xi_1,\Xi_2,\cdots,\Xi_n\right)=\left(\Xi_{o(1)}\right)^{\varpi_1}\oplus_{SW}\left(\Xi_{o(2)}\right)^{\varpi_2}\oplus_{SW}\cdots\oplus_{SW}\left(\Xi_{o(n)}\right)^{\varpi_n}, \quad (14)$$

where FFSWOWG is called as Fermatean fuzzy Sugeno-Weber ordered weighted averaging operator. $\left(o(1),o(2),\cdots,o(n)\right)$ is a permutation of $\left(1,2,\cdots,n\right)$ such that $\Xi_{o(j-1)}\geq\Xi_{o(j)},\ \forall j=2,3,\cdots,n$.

Theorem 5. The aggregation outcome of the set of FFNs $\Xi_j\left(j=1(1)n\right)$ utilizing FFSWOWG operator is an FFN and depicted as

$$FFSWOWG\left(\Xi_1,\Xi_2,\cdots,\Xi_n\right)=$$
$$\left(\sqrt[3]{\frac{1}{\Re}\left(\left(1+\Re\right)\prod_{j=1}^{n}\left(\frac{\Re\left(\kappa_{o(j)}\right)^3+1}{1+\Re}\right)^{\varpi_j}-1\right)},\sqrt[3]{\frac{1+\Re}{\Re}\left(1-\prod_{j=1}^{n}\left(1-\left(\xi_{o(j)}\right)^3\left(\frac{\Re}{1+\Re}\right)\right)^{\varpi_j}\right)}\right). \quad (15)$$

It is obviously that the FFSWWG and FFSWOWG operators also has the same properties with FFSWWA operator, such as idempotency, monotonicity, and boundedness

4. The proposed Fermatean fuzzy MCGDM approach

The section shall propose a novel MCGDM approach based on coefficient of variation method, WASPAS method and the proposed aggregation operators under Fermatean fuzzy setting. The proposed Fermatean fuzzy Sugeno-Weber operators are used to integrate the Fermatean fuzzy assessment information provided by decision experts. Next, the coefficient of variation method is propounded based on the Fermatean fuzzy distance measure and operator to estimate the importance of assessment criteria. Lastly, the improved WASPAS method is put forward based on the proposed FFSWWA and FFSWWG operators to attain the sorting of alternatives. The detailed procedures of the proposed MAGDM approach are depicted as follows:

Step 1: Construct the assessment matrix.

It supposed that a MAGDM problem includes the following notions. $G=\left\{G_i\left|i=1,2,\cdots,m\right.\right\}$ and $C=\left\{C_j\left|j=1,2,\cdots,n\right.\right\}$ denote the set of alternative and assessment criteria. ϖ_j is the weight of criteria C_j with the condition $\varpi_j\in[0,1],\sum_{j=1}^{n}\varpi_j=1$. Several experts from diverse files are invited to form as the evaluation committee denoted as $E=\left\{e_l\left|l=1,2,\cdots,L\right.\right\}$. ϕ_l denotes the importance of expert e_l and satisfies $\phi_l\in[0,1],\sum_{l=1}^{L}\phi_l=1$. Let $Q^{(l)}=\left(q_{ij}^{(l)}\right)_{m\times n}$ be an Fermatean fuzzy decision matric, where $q_{ij}^{(l)}$ signifies the assessment of scheme G_i with respect to criteria C_j provided by evaluation expert e_l.

Step 2: Determine the weight of assessment expert.

It is assumed that $\Xi_l = (\xi_l, \kappa_l)\left(l = 1(1)L\right)$ is an FFN. Every expert estimates their importance by linguistic variable in Table 1, then the weight ϕ_l of expert e_l can be computed by:

$$\phi_l = \frac{\left(\phi^3 - \kappa^3 + 1\right)}{\sum_{l=1}^{L}\left(\phi^3 - \kappa^3 + 1\right)}. \tag{16}$$

Step 3: Integrate the assessment matrices of experts.

To acquire the group intuitionistic fuzzy matrix $Q = \left(q_{ij}\right)_{m\times n}$ the assessment information of experts is fused by utilizing the FFSWWA operator or FFSWWG operator, wherein

$$q_{ij} = \left(\xi_{ij}, \kappa_{ij}\right) = FFSWWA\left(q_{ij}^{(1)}, q_{ij}^{(2)}, \cdots, q_{ij}^{(L)}\right)$$
$$= \left(\sqrt[3]{\frac{1+\Re}{\Re}\left(1 - \prod_{l=1}^{L}\left(1 - \left(\xi_{ij}^{(l)}\right)^3\left(\frac{\Re}{1+\Re}\right)\right)^{\phi_l}\right)}, \sqrt[3]{\frac{1}{\Re}\left(\left(1+\Re\right)\prod_{l=1}^{L}\left(\frac{\Re\left(\kappa_{ij}^{(l)}\right)^3 + 1}{1+\Re}\right)^{\phi_l} - 1\right)}\right). \tag{17a}$$

$$q_{ij} = \left(\xi_{ij}, \kappa_{ij}\right) = FFSWWG\left(q_{ij}^{(1)}, q_{ij}^{(2)}, \cdots, q_{ij}^{(L)}\right)$$
$$= \left(\sqrt[3]{\frac{1}{\Re}\left(\left(1+\Re\right)\prod_{l=1}^{L}\left(\frac{\Re\left(\xi_{ij}^{(l)}\right)^3 + 1}{1+\Re}\right)^{\phi_l} - 1\right)}, \sqrt[3]{\frac{1+\Re}{\Re}\left(1 - \prod_{l=1}^{L}\left(1 - \left(\kappa_{ij}^{(l)}\right)^3\left(\frac{\Re}{1+\Re}\right)\right)^{\phi_l}\right)}\right). \tag{17b}$$

Step 4: Determine the normalized group intuitionistic fuzzy matrix $\overline{Q} = \left(\overline{q}_{ij}\right)_{m\times n}$ with the help of the following equation:

$$\overline{q}_{ij} = \begin{cases} \left(\xi_{ij}, \kappa_{ij}\right), \text{benefit criteria} \\ \left(\kappa_{ij}, \xi_{ij}\right), \text{cost criteria} \end{cases}. \tag{18}$$

Step 5: Identify the weight of criteria via the Fermatean fuzzy coefficient of variation method. Based on the Fermatean fuzzy score function, the normalized group intuitionistic fuzzy matrix is shifted as score function matrix $\Box = \left(SI_{ij}\right)_{m\times n}$. Then, the mean value of the *j* th criteria is computed as:

$$\overline{\Box}_j = \frac{1}{m}\sum_{i=1}^{m}\left(SI_{ij}\right) \tag{19}$$

Step 6: To compute the standard deviation of the *j* th criteria by:

$$SD_j = \sqrt{\frac{1}{m-1}\sum_{i=1}^{m}\left(SI_{ij} - \overline{\Box}_j\right)^2},\ j = 1, 2, \cdots, n. \tag{20}$$

Step 7: To compute the coefficient of variation of the *j* th criteria;

$$X_j = \frac{SD_j}{\overline{\Box}_j},\ j = 1, 2, \cdots, n. \tag{21}$$

Step 8: Obtain the final weight of criteria after normalizing the coefficient of variation of each criteria.

$$\varpi_j = \frac{X_j}{\sum_{j=1}^{n} X_j}. \tag{22}$$

Step 9: Compute the additive relative importance of alternative based on the FFSWWA operator, wherein:

$$Q_i^1 = FFSWWA\left(q_{i1}, q_{i2}, \cdots, q_{in}\right)$$
$$= \left(\sqrt[3]{\frac{1+\Re}{\Re}\left(1-\prod_{j=1}^{n}\left(1-\left(\xi_{ij}\right)^3\left(\frac{\Re}{1+\Re}\right)\right)^{\phi_l}\right)}, \sqrt[3]{\frac{1}{\Re}\left(\left(1+\Re\right)\prod_{j=1}^{n}\left(\frac{\Re\left(\kappa_{ij}\right)^3+1}{1+\Re}\right)^{\phi_l}-1\right)} \right). \tag{23}$$

Step 10: Compute the multiplicative relative importance of alternative based on the FFSWWG operator, wherein:

$$Q_i^2 = FFSWWG\left(q_{i1}, q_{i2}, \cdots, q_{in}\right)$$
$$= \left(\sqrt[3]{\frac{1}{\Re}\left(\left(1+\Re\right)\prod_{j=1}^{n}\left(\frac{\Re\left(\xi_{ij}\right)^3+1}{1+\Re}\right)^{\phi_l}-1\right)}, \sqrt[3]{\frac{1+\Re}{\Re}\left(1-\prod_{j=1}^{n}\left(1-\left(\kappa_{ij}\right)^3\left(\frac{\Re}{1+\Re}\right)\right)^{\phi_l}\right)} \right). \tag{24}$$

Step 11: Compute the synthesize relative importance of alternatives:

$$Q_i = \aleph SI\left(Q_i^1\right) + \left(1-\aleph\right) SI\left(Q_i^2\right), \quad \aleph \in [0,1]. \tag{25}$$

where $\aleph$ is a balance index, when $\aleph = 1$, WASPAS can reduce to WSM model when $\aleph = 0$, WASPAS can reduce to WPM model.

Step 12: Attain the ranking of alternatives based on the values of synthesize relative importance $Q_i \left(i = 1, 2, \cdots, m\right)$, the best optimal option has the highest value of Q_i.

5. The proposed Fermatean fuzzy MAGDM approach

The current section validates the feasibility of the proposed MAGDM approach by a reverse logistics supplier evaluation problem. The sensitivity analysis and comparison study are also executed to estimate the stability and superiority of the proposed group decision approach.

5.1 Background description

The rapid development of economic globalization has led to increasingly fierce competition among enterprises. In order to further enhance the core competitiveness of enterprises and improve their comprehensive development capabilities in a limited resource environment, the reverse logistics operation model has been widely promoted and gradually introduced into the logistics management of enterprises. Third party reverse logistics suppliers can not only concentrate the main industries of enterprises, but also reduce costs, decrease funds, and gradually improve their core competitiveness. However, due to insufficient understanding of reverse logistics suppliers and unfamiliarity with partners, it is difficult for enterprises to choose suitable reverse logistics suppliers. Therefore, determining suitable reverse logistics suppliers is of great strategic significance for the high-quality development of enterprises. Considering the complexity and fuzziness of human cognitive abilities, this paper determines the optimal reverse logistics supplier based on the proposed

Fermat fuzzy group decision-making method. The reverse logistics supplier evaluation indicators denoted as $\{C_1, C_2, C_3, C_4, C_5\}$ are determined by three experts through comprehensive research to provide evaluation opinions and determine the optimal reverse logistics supplier, the explanation of the selected criteria are depicted in Table 1. The management department of enterprise has selected four alternative reverse logistics suppliers denoted as $\{G_1, G_2, G_3, G_4, G_5\}$ through qualification review and expert evaluation. The aim of the management department of enterprise is to determine the optimal reverse logistics supplier to develop logistics transportation services.

Table 1
The explanation of the selected reverse logistics supplier assessment criteria

Criteria	Explanation
Reverse logistics costs (C_1)	It mainly includes transportation costs, warehousing costs, order costs, and packaging costs throughout the entire logistics process.
Service level (C_2)	It refers to the level of customer satisfaction, logistics center processing efficiency, and order delivery responsiveness in the process of reverse logistics management.
Informationization level (C_3)	It mainly refers to the security and timeliness of logistics information, on-time delivery of goods, information sharing and integration capabilities, and the proportion of computer professionals in the reverse logistics management process.
Technical level (C_4)	It mainly refers to the innovation ability of technical personnel in the process of reverse logistics management, the proportion of R&D investment in enterprises, the level of technical personnel, and the leading level of equipment.
Strategic alliance (C_5)	It mainly refers to the inclusiveness of the enterprise's strategic development concept, the compatibility of cultural level, the sharing of enterprise risks, and the communication ability between partners in the process of reverse logistics management.

5.2 Decision analysis procedure

In the following, we will use the proposed group decision approach to select the optimal reverse logistics supplier.

Step 1: The invited three experts use FFN to express their assessment for the alternative reverse logistics suppliers with respect to criteria, which are displayed in Table 2-4.

Table 2
The Fermatean fuzzy evaluation for RLSs provided by expert e_1

RLSs	C_1	C_2	C_3	C_4	C_5
G_1	(0.20, 0.85)	(0.85, 0.35)	(0.75, 0.25)	(0.70, 0.35)	(0.65, 0.50)
G_2	(0.25, 0.85)	(0.60, 0.40)	(0.85, 0.20)	(0.80, 0.30)	(0.60, 0.50)
G_3	(0.30, 0.80)	(0.65, 0.35)	(0.70, 0.30)	(0.65, 0.50)	(0.70, 0.55)
G_4	(0.15, 0.90)	(0.55, 0.45)	(0.65, 0.25)	(0.60, 0.50)	(0.75, 0.50)
G_5	(0.25, 0.75)	(0.55, 0.50)	(0.80, 0.25)	(0.75, 0.35)	(0.65, 0.45)

Table 3
The Fermatean fuzzy evaluation for RLSs provided by expert e_2

RLSs	C_1	C_2	C_3	C_4	C_5

G_1	(0.30, 0.80)	(0.65, 0.30)	(0.70, 0.30)	(0.80, 0.30)	(0.65, 0.45)
G_2	(0.30, 0.80)	(0.70, 0.30)	(0.55, 0.45)	(0.65, 0.25)	(0.60, 0.50)
G_3	(0.15, 0.90)	(0.85, 0.30)	(0.75, 0.25)	(0.70, 0.35)	(0.70, 0.30)
G_4	(0.25, 0.75)	(0.55, 0.50)	(0.80, 0.25)	(0.75, 0.35)	(0.80, 0.30)
G_5	(0.30, 0.80)	(0.65, 0.45)	(0.85, 0.20)	(0.80, 0.30)	(0.60, 0.50)

Table 4
The Fermatean fuzzy evaluation for RLSs provided by expert e_3

RLSs	C_1	C_2	C_3	C_4	C_5
G_1	(0.30, 0.80)	(0.80, 0.30)	(0.65, 0.30)	(0.70, 0.30)	(0.80, 0.30)
G_2	(0.30, 0.80)	(0.65, 0.25)	(0.60, 0.50)	(0.85, 0.30)	(0.75, 0.25)
G_3	(0.15, 0.90)	(0.80, 0.30)	(0.80, 0.30)	(0.65, 0.30)	(0.65, 0.25)
G_4	(0.15, 0.90)	(0.85, 0.30)	(0.75, 0.20)	(0.65, 0.25)	(0.60, 0.50)
G_5	(0.25, 0.75)	(0.70, 0.30)	(0.55, 0.45)	(0.65, 0.25)	(0.80, 0.30)

Step 2: Based on the experts' cognition ability, three experts estimate their importance by FFNs, then the weight of experts can be computed by Eq.(16), the results are shown as $\phi_1 = 0.3502$, $\phi_2 = 0.2969$, $\phi_3 = 0.3529$.

Step 3: With the aid of Eq.(17), the Fermatean fuzzy group assessment matrix can be attained by using FFSWWA operator, which is shown in Table 5.

Table 5
Fermatean fuzzy group assessment matrix

RLSs	C_1	C_2	C_3	C_4	C_5
G_1	(0.2731, 0.8178)	(0.7848, 0.3192)	(0.7031, 0.2844)	(0.7338, 0.3192)	(0.7130, 0.4301)
G_2	(0.2845, 0.8178)	(0.6504, 0.3294)	(0.7072, 0.4175)	(0.7850, 0.2869)	(0.6631, 0.4399)
G_3	(0.2269, 0.8661)	(0.7750, 0.3192)	(0.7534, 0.2869)	(0.6659, 0.4024)	(0.6834, 0.4113)
G_4	(0.1915, 0.8579)	(0.6972, 0.4269)	(0.7368, 0.2347)	(0.6694, 0.3936)	(0.7247, 0.4562)
G_5	(0.2669, 0.7652)	(0.6402, 0.4301)	(0.7557, 0.3466)	(0.7366, 0.3054)	(0.7021, 0.4269)

Step 4: According to the Eq.(18), the normalized Fermatean fuzzy group assessment matrix can be attained by shifted the cost-type criteria to benefit-type criteria, the result is shown in Table 6 .

Table 6
Normalized Fermatean fuzzy group assessment matrix

RLSs	C_1	C_2	C_3	C_4	C_5
G_1	(0.8178, 0.2731,)	(0.7848, 0.3192)	(0.7031, 0.2844)	(0.7338, 0.3192)	(0.7130, 0.4301)
G_2	(0.8178, 0.2845)	(0.6504, 0.3294)	(0.7072, 0.4175)	(0.7850, 0.2869)	(0.6631, 0.4399)
G_3	(0.8661, 0.2269)	(0.7750, 0.3192)	(0.7534, 0.2869)	(0.6659, 0.4024)	(0.6834, 0.4113)

G_4	(0.8579, 0.1915)	(0.6972, 0.4269)	(0.7368, 0.2347)	(0.6694, 0.3936)	(0.7247, 0.4562)
G_5	(0.7652, 0.2669)	(0.6402, 0.4301)	(0.7557, 0.3466)	(0.7366, 0.3054)	(0.7021, 0.4269)

Step 5-8: Based upon the Fermatean fuzzy score function, the normalized group intuitionistic fuzzy matrix is shifted as score function matrix. Then, the mean value of the *j* th criteria is computed by Eq.(19), namely, $\square_1 = 0.7742$, $\square_2 = 0.6567$, $\square_3 = 0.6787$, $\square_1 = 0.6663$, $\square_5 = 0.6293$. Then the standard deviation of the *j* th criteria is attained by Eq.(20) and the coefficient of variation of the *j* th criteria is attained by Eq.(21). Lastly, the weight of criteria can be identified by Eq.(22), the calculation results can be shown as: $\varpi_1 = 0.1935$, $\varpi_2 = 0.3233$, $\varpi_3 = 0.1360$, $\varpi_4 = 0.2566$, $\varpi_5 = 0.0906$.

Step 9: The additive relative importance of ELSs based on the FFSWWA operator shown in Eq.(23), the calculation results can be shown as: $Q_1^1 = (0.7636, 0.3208)$, $Q_2^1 = (0.7352, 0.3404)$, $Q_3^1 = (0.7630, 0.3385)$, $Q_4^1 = (0.7382, 0.3733)$, $Q_5^1 = (0.7152, 0.3662)$.

Step 10: The multiplicative relative importance of ELSs based on the FFSWWG operator shown in Eq.(24), the calculation results can be shown as: $Q_1^2 = (0.7617, 0.3214)$, $Q_2^2 = (0.7300, 0.3414)$, $Q_3^2 = (0.7570, 0.3395)$, $Q_4^2 = (0.7324, 0.3751)$, $Q_5^2 = (0.7127, 0.3674)$.

Step 11: The synthesize relative importance of ELSs are compute by Eq.(25) and shown as: $Q_1 = 0.7052$, $Q_2 = 0.6768$, $Q_3 = 0.7000$, $Q_4 = 0.6726$, $Q_5 = 0.6573$.

Step 12: Based on the values of synthesize relative importance, the ranking of alternative RLSs can be attained as. $G_1 \succ G_3 \succ G_2 \succ G_4 \succ G_5$, namely, the optimal RLSs is the first RLS (G_1).

5.3 Sensitivity analysis

The stability and robustness of the decision approach is important for managers to utilize the proposed method to realistic decision analysis. In the proposed Fermatean fuzzy group decision approach, we discuss the influence of parameter $\Re$ for the final ranking of alternative RLSs. Based on the range of parameter $\Re$, we select ten diverse parameter values of $\Re$ and use the propose method to attain the comprehensive assessment grade of alternative RLSs and their ranking, the results are shown in Figure 1**.** Frome the acquired outcomes, we can observe that changes in parameters have no effect on the final ranking of RLS, indicating that the proposed method has extremely high stability when parameters change.

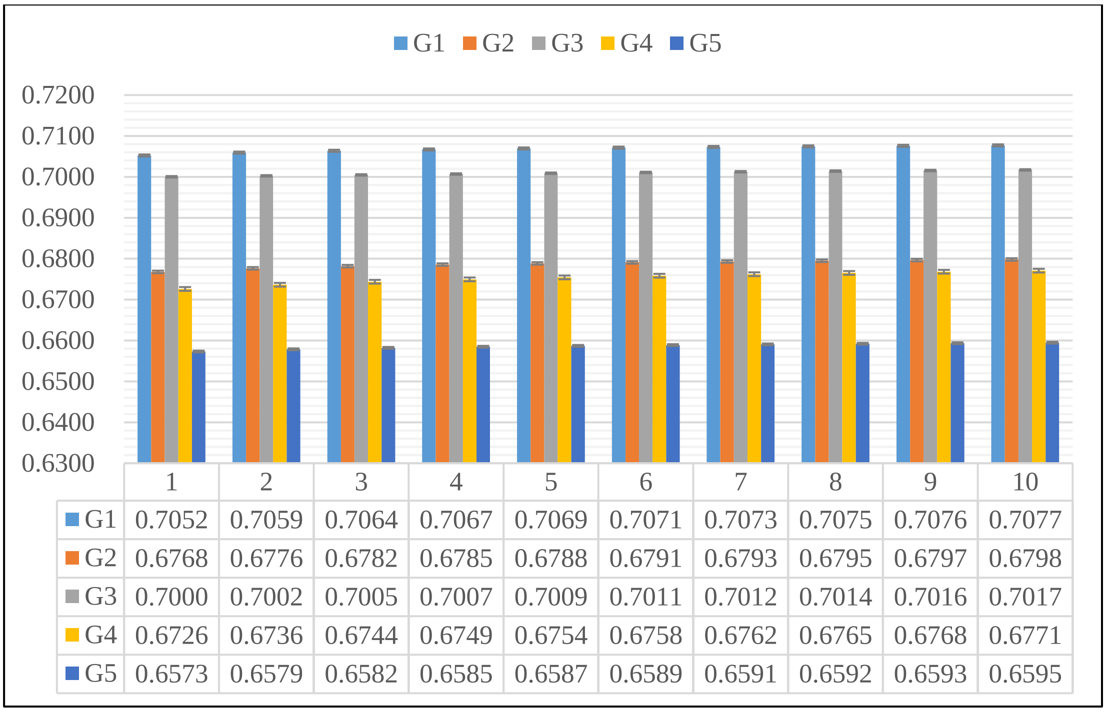

Fig. 1. The synthesize relative importance of alternative ELSs under diverse parameter $\mathfrak{R}$

In addition, the balance parameter in WASPAS approach maybe influence the ranking of RLSs. Hence, we select different parameter value of $\aleph$ and attain the corresponding ranking of RLSs, the results are shown in Table 7 and Figure 2. From Table, we can observe that the ranking results do not change when changing the parameter value. Accordingly, the parameter $\aleph$ will not have any impact on the final sorting value, indicating that the proposed method has strong stability.

Table 7
The comprehensive assessment grade of RLSs under diverse parameter $\aleph$

Parameter $\aleph$	**Comprehensive relative importance**					**Ranking**
	G1	G2	G3	G4	G5	
0.1	0.7045	0.6751	0.6979	0.6706	0.6564	$G_1 \succ G_3 \succ G_2 \succ G_4 \succ G_5$
0.2	0.7047	0.6755	0.6984	0.6711	0.6567	$G_1 \succ G_3 \succ G_2 \succ G_4 \succ G_5$
0.3	0.7049	0.6759	0.6989	0.6716	0.6569	$G_1 \succ G_3 \succ G_2 \succ G_4 \succ G_5$
0.4	0.7050	0.6763	0.6995	0.6721	0.6571	$G_1 \succ G_3 \succ G_2 \succ G_4 \succ G_5$
0.5	0.7052	0.6768	0.7000	0.6726	0.6573	$G_1 \succ G_3 \succ G_2 \succ G_4 \succ G_5$
0.6	0.7054	0.6772	0.7006	0.6731	0.6575	$G_1 \succ G_3 \succ G_2 \succ G_4 \succ G_5$
0.7	0.7056	0.6776	0.7011	0.6736	0.6577	$G_1 \succ G_3 \succ G_2 \succ G_4 \succ G_5$
0.8	0.7058	0.6781	0.7016	0.6741	0.6580	$G_1 \succ G_3 \succ G_2 \succ G_4 \succ G_5$
0.9	0.7059	0.6785	0.7022	0.6746	0.6582	$G_1 \succ G_3 \succ G_2 \succ G_4 \succ G_5$
1.0	0.7061	0.6789	0.7027	0.6752	0.6584	$G_1 \succ G_3 \succ G_2 \succ G_4 \succ G_5$

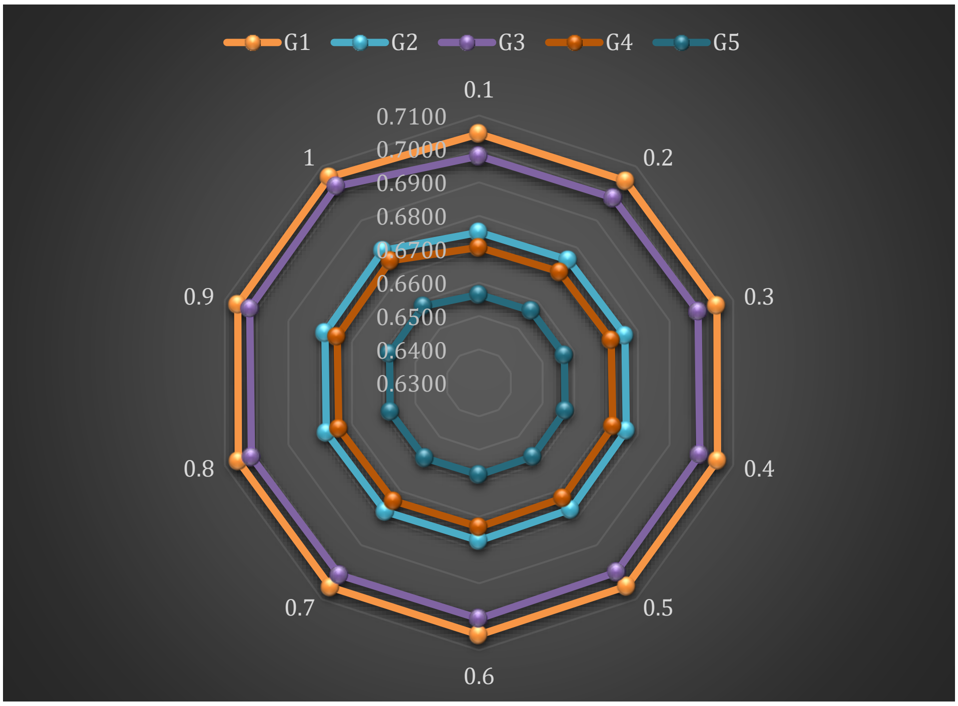

Fig. 2. The Comprehensive assessment grade of alternative RLSs under diverse parameter $\aleph$

5.4 Comparative study

To confirm the effectiveness and feasibility of the proposed approaches, we implement a comparison analysis between the proposed approach and the existing methods including FF-WPM method[29], FF-ARAS method[28], FFWA operator-based method[22]. The comparative results are displayed in Table 8 and Figure 3.

Table 8
Comparison outcomes between the existing and the presented methods

Approach	Comprehensive assessment grade					**Ranking**
	G1	G2	G3	G4	G5	
FF-WPM method **[29]**	0.5322	0.5160	0.4900	0.4710	0.5216	$G_1 \succ G_5 \succ G_2 \succ G_3 \succ G_4$
FF-ARAS method**[28]**	0.9154	0.8651	0.9101	0.8799	0.8363	$G_1 \succ G_3 \succ G_4 \succ G_2 \succ G_5$
FFWA operator-based method[22]	0.6506	0.6217	0.6310	0.6002	0.6080	$G_1 \succ G_3 \succ G_2 \succ G_5 \succ G_4$
Presented approach	0.7052	0.6768	0.7000	0.6726	0.6573	$G_1 \succ G_3 \succ G_2 \succ G_4 \succ G_5$

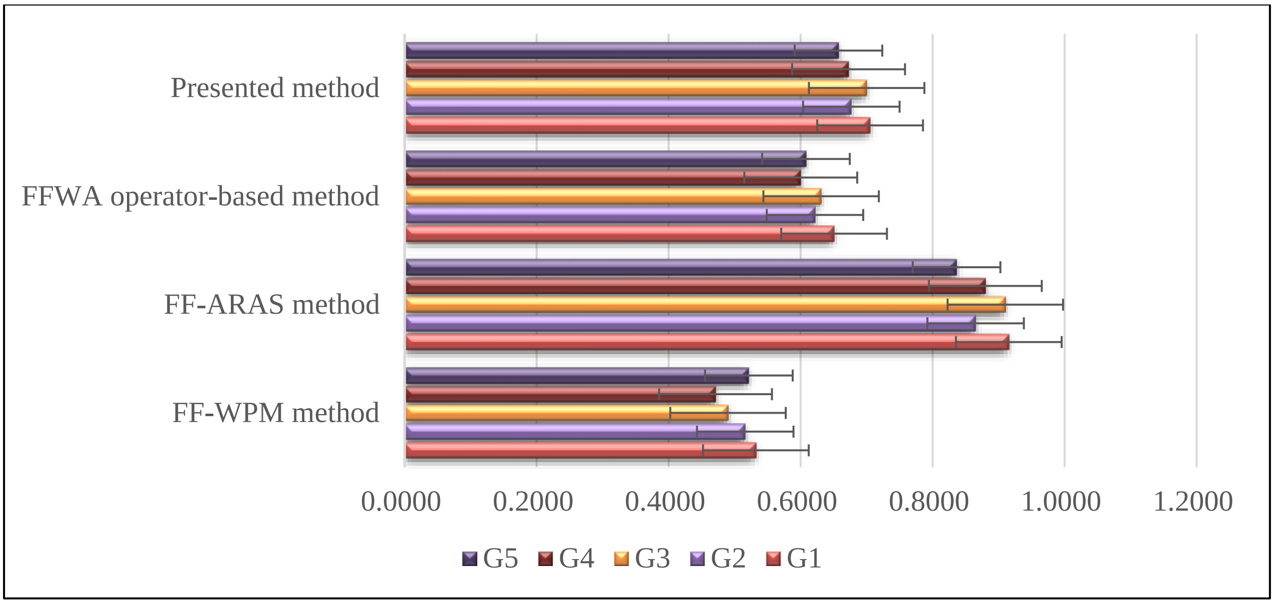

Fig. 3. Comparison outcomes between the existing approach and the presented method

From the comparative outcome, we can find that the ranking attained by the existing approaches has a little difference with the sorting obtained by the proposed Fermatean fuzzy group decision approach. However, the optimal option determined by these methods are all the first alternative RLS, which signifies the validity of the propounded approach. Further, the comparison analysis of the proposed approach can be summarized as follows:

(1) The proposed group decision approach under Fermatean fuzzy setting provides a universal decision framework for decision makers to select the suitable RLS without weight providing weight information;

(2) In the stage of information fusion, the proposed approach proposed utilize the FFSWWA operator to integrate the assessment information of experts, which make the decision procedure possesses stronger robustness. Further, the parameter can be viewed as the risk preference parameter of expert to flexibly unfold decision analysis.

(3) The WASPAS method is improved based on the Fermatean fuzzy Sugeno-Weber operators to determine the ranking of alternatives, which makes the decision process more stability.

6. Conclusion

The assessment and selection of reverse logistics suppliers is a critical initiative for enterprises aiming to enhance their core competitiveness. In response, this study proposes a novel Multi-Attribute Group Decision-Making (MAGDM) methodology tailored for selecting reverse logistics suppliers using Fermatean fuzzy information. To effectively aggregate Fermatean fuzzy information, Sugeno-Weber operations are extended to the Fermatean fuzzy environment, and four new Fermatean fuzzy Sugeno-Weber aggregation operators are introduced. Additionally, a Fermatean fuzzy coefficient of variation method is developed to determine the importance weights of assessment criteria. Furthermore, an improved Weighted Aggregated Sum Product Assessment (WASPAS) method, based on the proposed operators, is presented to prioritize alternatives.A case study is conducted to demonstrate the applicability of the proposed MAGDM methodology. Sensitivity analysis and comparative studies are performed to evaluate the stability and superiority of the proposed approach. Looking ahead, future research directions include: (1) extending Sugeno-

Weber operations to other uncertain and fuzzy environments, such as complex q-rung orthopair fuzzy sets, linear Diophantine fuzzy sets, and linguistic q-rung orthopair fuzzy sets; (2) developing additional novel aggregation operators by integrating Sugeno-Weber operations with power averaging operators, softmax functions, Muirhead means, and other techniques; (3) incorporating consensus processes to improve the rationality of group decision-making methodologies; and (4) exploring alternative methodologies by combining novel decision algorithms, such as Weight by Envelope and Slope (WENSLO), Alternative Ranking using Two-Step Logarithmic Normalization (ARLON), and generalized TODIM methods. These future directions aim to further advance the field of decision-making under uncertainty.

Author Contributions

"Conceptualization, Y.R. and G.W.; methodology, Y.R. and G.W.; validation, Y.R.; formal analysis, Y.R.; writing—original draft preparation, Y.R. and G.W.; writing—review and editing. All authors have read and agreed to the published version of the manuscript." Authorship must be limited to those who have contributed substantially to the work reported.

Data Availability Statement

The data used to support the findings of this study are included within this article. However, the reader may contact the corresponding author for more details on the data.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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