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Digital Twin Technology in Teaching Geometry: Didactic Potential and a Methodology for Visualizing the Concepts of Area and Diameter

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ABSTRACT

The article considers the possibilities of using digital resources and technologies of digital twins in the process of teaching geometry, in particular when forming ideas about the concepts of area and diameter. Digital twins are virtual models of real geometric objects that reproduce their properties, parameters and relationships in a dynamic mode. This approach allows students not only to observe the figure, but also to study its behavior when changing individual parameters, which contributes to a deeper understanding of mathematical patterns. The didactic potential of interactive learning tools, in particular GeoGebra, Desmos, PhET, is substantiated. Interactive Simulations and LearningApps, which can perform the functions of digital twins of geometric objects. Their use ensures the creation of a dynamic educational environment in which students have the opportunity to model circles, measure diameters, calculate areas, and also observe in real time the changes that occur when varying the radius or scale of a figure. It is shown that the technology of digital twins promotes interactive and research learning, activates the cognitive activity of students, develops spatial thinking and the ability to apply knowledge in practice. The article identifies methodological approaches to the implementation of digital twins in geometry teaching, including the creation of virtual laboratories, interactive experiments and tasks with parametric changes of figures. Special attention is paid to the role of digital twins as a tool for individualizing learning, which allows adapting the complexity of tasks and visualizing the process of forming geometric concepts at different levels of cognition. The use of such technologies meets the requirements of STEM education and the concept of the New Ukrainian School, where digital competence is considered a key component of modern mathematics education. Thus, the use of digital twins in teaching geometry not only expands the possibilities of visualizing the concepts of area and diameter, but also forms a new quality of the educational process – exploratory, interactive, and competency-oriented.

1. Introduction

The modern education system is in a state of profound transformation, caused by the active introduction of digital technologies into all spheres of human activity – from science and production

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to social communication and everyday life. The digitalization of education today is considered not only as a technical update of the educational process, but as a strategic change in the educational paradigm, the focus of which is the formation of a student who is able to think critically, creatively apply knowledge and effectively interact with the digital environment. In this context, twenty-first century education is increasingly focused on the development of digital, technological and information competencies.

One of the key areas of digitalization of the educational process is the introduction of interactive digital resources and digital twin technologies (digital twins), which open up new possibilities for visualization, modeling, experimentation, and personalization of learning. Digital twin technology is not just a computer model, but a dynamic, self-learning digital copy of a real object or process, capable of reproducing its behavior under changing conditions. In a pedagogical context, a digital twin can be represented as a virtual analogue of a learning object, allowing students to interact with it in real time, experiment with parameters, observe the consequences of their actions, and draw scientifically based conclusions.

Geometry teaching is traditionally based on the principle of visualization, which involves the use of models, drawings and illustrations to form spatial thinking and an idea of the properties of shapes. However, modern pedagogy is moving from passive visualization to active digital interaction, when the student does not just contemplate a finished shape, but independently constructs, modifies and analyzes it. This transition from static images to dynamic interactive models allows not only to increase the level of knowledge acquisition, but also to form an investigative style of thinking, which is an important component of STEM education.

It is the technology of digital twins that provides the opportunity to create such living geometric models that integrate mathematical accuracy, visual appeal and adaptability to the individual characteristics of students. In the context of studying the concepts of area and diameter, digital twins allow in real time to explore the relationships between the parameters of a circle, change the diameter, radius, scale and observe how these changes affect the area of the figure. This allows students to comprehend functional relationships in geometry not only formally, but also empirically, through digital experimentation.

Another advantage of digital twins is their ability to combine mathematical modeling with practical actions. Students can test hypotheses, create their own variations of problems, and compare results with analytical calculations. This approach promotes the development of critical, algorithmic, and spatial thinking, forms skills in working with digital data, and develops the ability to make decisions based on observations and analysis.

The relevance of the study is due to the need to update the methodology of teaching geometry in accordance with the requirements of STEM education, the Concept of the New Ukrainian School (NUS) and the Competency Approach in Education, which are focused on practicality, interdisciplinarity and active use of information and communication technologies. Although there are currently a large number of digital tools for teaching mathematics – GeoGebra, Desmos, LearningApps, PhET Simulations, Matific, etc., – their use is mostly limited to an illustrative function. The issue of creating full-fledged digital twins of geometric objects, which allow reproducing real processes of measuring, constructing and transforming figures in the educational environment, still remains insufficiently researched and requires systematic scientific and methodological understanding.

2. Literature review

The issue of using digital technologies in teaching mathematics and geometry has gained particular importance in the context of the digital transformation of education. Modern researchers [1–3] note that the digital learning environment not only contributes to the formation of new cognitive strategies, but also acts as a means of rethinking pedagogical interaction, creating conditions for the transition from reproductive to research-based learning. In this context, technologies serve not just as a tool, but as a factor that restructures the student's thinking, activating analytical, visual and algorithmic mechanisms of cognition.

A detailed analysis of the scientific literature shows that the issue of digital visualization and modeling of geometric concepts remains central to pedagogical research in the last decade. Tesfamicael [4], Karassyova et al. [5] and Glaze et al. [6] emphasize that dynamic environments, such as GeoGebra or Desmos, not only illustrate mathematical dependencies, but also create conditions for the development of metacognitive skills – the ability to analyze, check and correct one's own understanding. A comparative analysis of the works of these authors gives grounds to argue that it is the possibility of instantly changing parameters and observing the result that forms students' understanding of cause-and-effect relationships in mathematical objects.

Studies by Rahimi and Oh [7] and Geng [8] show a tendency to rethink the role of the learner in the digital environment. The concept of dynamic geometry in these works appears as a tool for democratizing learning – the learner becomes not a consumer, but a creator of mathematical models. This approach is consistent with the constructivist ideas of Nasr et al. [9], who viewed the computer as a "thinking partner" that enables the implementation of the principle of learning through action and exploration.

Latest works by Hellín et al. [10] and Abdulbaki et al. [11] emphasize another function of interactive platforms – motivational. LearningApps, PhET Interactive Simulations, Matific, and other online tools create a gamified learning environment where every action has immediate feedback and the learner experiences self-regulation. In this regard, digital resources become part of a formative assessment ecosystem that ensures learning is adapted to individual educational trajectories.

A separate vector of research is related to the implementation of digital twins (digital twins). The first theoretical foundations of this approach were laid in engineering sciences [12, 13], however, in the twenty-first century the concept began to actively develop in the pedagogical sphere. Budanov et al. [14] and Brovko et al. [15] demonstrate that digital twins allow for the creation of virtual laboratories where students can experiment with model parameters in real time. Such simulations significantly expand the possibilities of STEM education, especially in the aspect of studying spatial and dynamic processes.

In the Ukrainian scientific and educational context [16, 17] the idea of digital twins is seen as an important tool for the development of interdisciplinary thinking. Geometry in this case appears not as an isolated field, but as part of an integrated environment where students can combine mathematical knowledge with engineering, technological and artistic aspects. Such approaches correlate with modern models of STEAM education, in which the educational process is based on project-based research activities, critical analysis and creative construction of knowledge.

At the same time, the analysis of modern sources indicates a certain scientific gap – the lack of systematic research on the methodology for creating and integrating digital twins of geometric figures into the school course. Clear criteria for the pedagogical effectiveness of such tools have not been developed, there is a lack of empirical data on their impact on the depth of understanding of the concepts of "area", "diameter", "symmetry". Accordingly, there is a need for an interdisciplinary approach that combines pedagogical theory, cognitive psychology and computer science.

Thus, the current state of scientific development of the problem indicates the active formation of a new direction – pedagogical digital geometry, within which digital twins can become an innovative means of learning that integrates technological, analytical and creative aspects of cognition.

3. Materials and methods

The study is of a theoretical and analytical nature and is aimed at determining the didactic potential of digital resources in the process of visualizing the concepts of area and diameter during the study of geometry in the context of the Concept of the National Secondary School. The main attention is paid to the generalization of modern theoretical approaches, the analysis of digital environments and the substantiation of the concept of a digital twin of a geometric object as an innovative means of learning.

The work used methods of theoretical analysis, synthesis, comparison, generalization and systematization of scientific sources, which made it possible to identify the main trends in the development of digital education, determine the directions of implementing visualization technologies in the school geometry course and outline the didactic conditions for their effective application. Analysis of the works of Ukrainian and foreign researchers [18–20] showed that the introduction of digital technologies into learning changes the very nature of mathematical thinking, making it visual-exploratory.

The comparative study of digital platforms was carried out according to the criteria of interactivity, clarity, the ability to control parameters, support for three-dimensional visualization and pedagogical integration. The results of the analysis confirmed that the most effective for the visualization of geometric concepts are dynamic environments in which the student can observe the relationship between changing the parameters of the figure and the result, which creates conditions for the formation of cause-and-effect thinking.

The study theoretically substantiates the concept of a digital twin of a circle – an interactive model that allows exploring the relationship between radius, diameter and area in real time. This approach contributes to the transition from abstract learning of formulas to a deep visual-analytical understanding of mathematical patterns. To describe the functioning of the digital twin, a systemic and structural-functional approach was used, which allowed us to determine its main educational functions: didactic – ensuring understanding of spatial relationships; motivational – increasing interest in learning through interactivity; cognitive research – creating conditions for independent discovery of mathematical relationships.

As a result of theoretical modeling, a general sequence of creating a digital twin was developed, which involves defining the educational goal, describing functional elements, building interactive visualization, and forming methodological recommendations for integrating the model into the educational process. The research materials were scientific sources, open educational platforms, methodological documents of the Ministry of Education and Science of Ukraine, and examples of practical use of digital environments in teaching geometry.

The results obtained indicate that the creation of a digital twin of a geometric object is not only a technical, but primarily a pedagogically justified process that contributes to the development of a new direction in digital pedagogy – pedagogical digital modeling. This approach combines didactic principles, technological capabilities and cognitive mechanisms for the formation of mathematical representations, providing a qualitatively new level of visualization and comprehension of educational material.

4. Results

In the process of theoretical analysis, it was found that the use of digital resources in teaching geometry is one of the most dynamic areas of modern pedagogical science. Digital technologies not only change the teaching tools, but also affect the cognitive structure of mathematical knowledge, making it more visual, interactive and research-based. Research of scientific sources [21, 22] show that the digital learning environment contributes to the formation of a new type of mathematical thinking – visual-algorithmic, within which the student is able not only to perceive ready-made images or graphic models, but also to actively construct his own ideas about mathematical objects.

In particular, the analysis of the works of Rizos and Gkrekas [23] and Yohannes and Chen [24] allows us to conclude that the dynamic geometry environment (e.g., GeoGebra, Desmos) creates unique conditions for the assimilation of concepts related to spatial and metric relations. The student has the opportunity to observe how a change in one parameter automatically affects others – for example, how a change in the diameter of a circle leads to a proportional change in its area. This form of representation of mathematical dependencies provides a visual understanding of the formula for the area of a circle ($S = \pi r^{2)}$, which is fundamentally important for understanding it not as an abstract expression, but as a pattern that has visual confirmation.

Researchers Almpani et al. [25] and Brendefur et al. [26] note that interactive digital environments create conditions for a gradual transition from empirical observation to formalized mathematical thinking. When a student independently changes the parameters of a figure, measures the area or distance, observes the relationships between elements, cause-and-effect relationships are actively formed, which increases the depth of understanding of concepts. Thus, digital visualization makes it possible to integrate practical experience with theoretical knowledge, which is often lacking in traditional geometry teaching.

The analysis of foreign and Ukrainian sources also showed that the effectiveness of digital tools depends not only on their technical characteristics, but also on the didactic scenario of use. An interactive platform in itself does not guarantee a learning effect – the decisive factor is how the teacher integrates it into the structure of the lesson, what tasks are set for students, how reflection is organized. Therefore, digital resources should be considered as a pedagogical system in which the software tool plays the role of a visualization tool, and not an end in itself.

The generalization of modern research allows us to highlight a number of main trends that determine the current state of the problem. First, there is a transition from a static presentation of educational material to a dynamic presentation of mathematical objects, which increases the level of cognitive activity of students. Second, there is a shift in emphasis from the process of knowledge reproduction to research interaction with digital models. Third, a new type of learning environment is being formed, in which visualization and simulation play a leading role in the formation of meaningful representations.

Particular attention during the analysis was paid to the problem of visualizing the concepts of area and diameter, which are basic, but are traditionally perceived by students as abstract. Digital resources allow us to move from a verbal-formulaic to an operational understanding of these concepts. For example, in GeoGebra it is possible not only to construct a circle and measure its parameters, but also to create a model that shows the relationship between area and diameter in real time. Such visualization provides a double effect – cognitive (the student better understands the meaning of the mathematical relationship) and motivational (interest in learning increases, since the abstract formula "comes to life").

The MATLAB environment is one of the most convenient tools for creating educational visualizations of mathematical concepts, as it combines means of calculation, graphical display and interactive user interaction with the model. Thanks to built-in functions for constructing geometric objects, creating graphical interfaces and using MATLAB programming code, the teacher can easily implement dynamic demonstrations that help students better understand the essence of mathematical dependencies.

In the process of learning geometry, MATLAB allows modelling basic concepts such as circle, radius, diameter, and area, with the ability to visually display them in the form of interactive figures. Using control elements (sliders, buttons, text fields), students can independently change the parameters of the construction, observe how the diameter and area automatically change when the radius increases, and compare the results with analytical expressions. In this way, not only an abstract idea of the mathematical formula for the area of a circle is formed, but also a visual understanding of the relationship between quantities.

Using tools for creating graphical interfaces (uifigure, uigridlayout, uiaxes), the teacher can organize the learning space so that a circle is displayed in the center of the screen, and explanations, formulas and interaction elements are located at the bottom. MATLAB allows controlling colors, line thickness, transparency of objects, which creates the opportunity to aesthetically design educational demonstrations. For the convenience of the educational process, a function of automatic saving of screenshots is provided, which allows to record the results of the study and use them when preparing reports or presentations. Figure 1 shows an example of a created interface in which the student changes the value of the radius of a circle, and the program instantly calculates the area and diameter, displaying them in a colored graphics window.

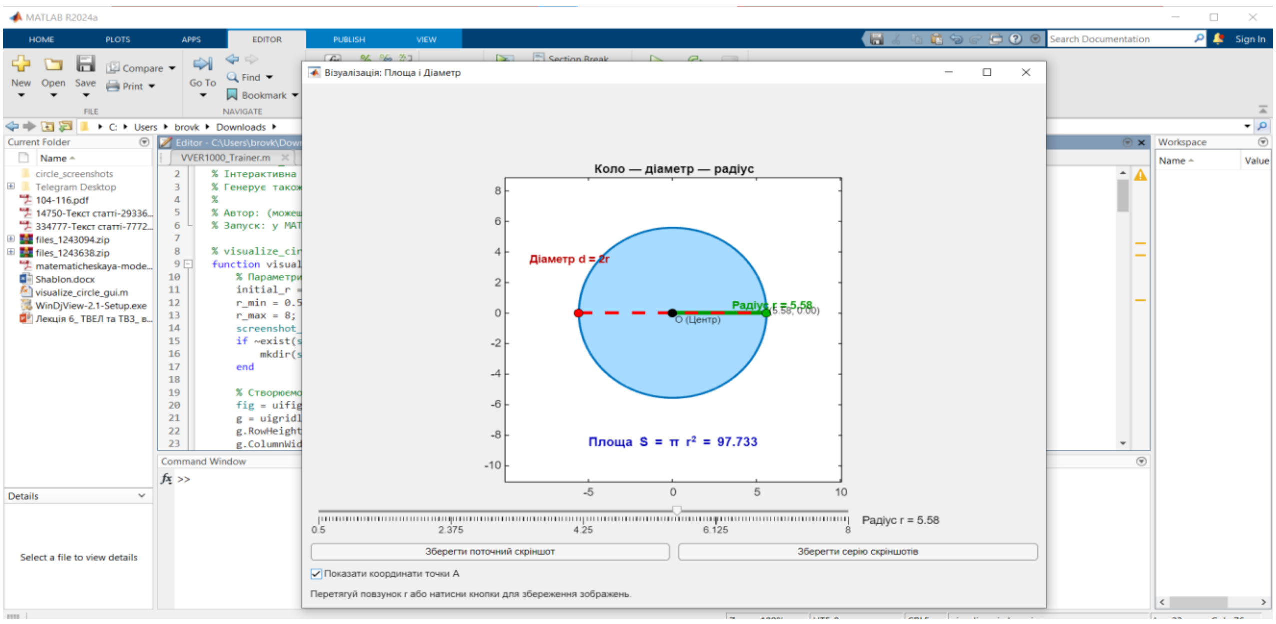

Fig. 1. The interface of the interactive model in MATLAB: on the left – a graphical representation of a circle with the radius (blue) and diameter (green) marked; on the right – a control panel with a slider for changing the radius and displaying the calculated values of the area and diameter
Source: complied by authors

This approach provides a combination of practical activities with visual thinking, which contributes to a deeper understanding of geometric concepts. MATLAB is not just a computing tool, but a full-fledged educational tool that allows to create interactive digital resources to support the educational process.

Blender allows to create interactive and visual models of geometric objects, making it an effective tool for teaching mathematical and physical concepts. In particular, Blender and the Python programming language can model three-dimensional objects, such as spheres, and display key characteristics on them: radius, diameter, surface area, and volume.

In the animation presented (Figure 2), the sphere gradually increases in size, demonstrating the dependence of the surface area and volume on the radius. For clarity, colored cylinders have been added to represent the radius and diameter, as well as black captions that explain the main quantities. This combination of graphics, animation, and text explanations allows students to simultaneously see the geometric image of the object and the mathematical formulas that describe it.

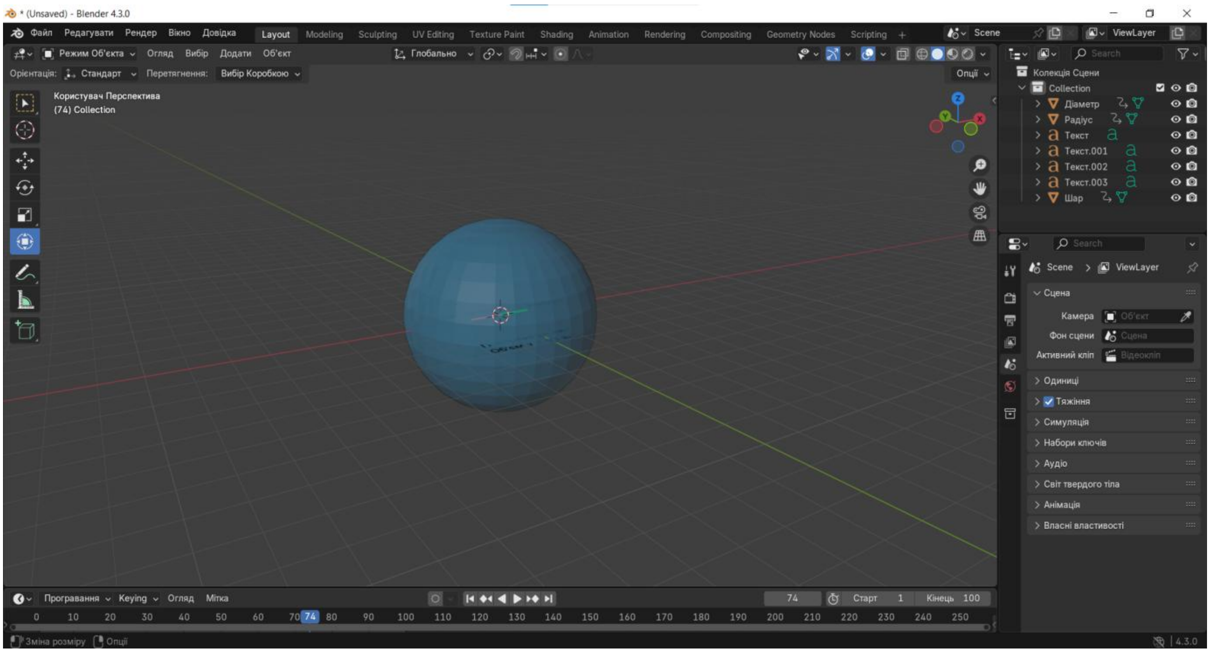

Fig. 2. Visualization of a sphere in Blender: gradually increasing the radius demonstrates the dependence of surface area and volume on radius; colored cylinders indicate radius and diameter, black inscriptions are key quantities
Source: complied by authors

The program provides interactivity in the learning process: a teacher or student can change the parameters of the sphere, observe how the related quantities change, and take screenshots for further analysis. The use of black for captions increases contrast and makes the information easy to perceive even on projectors or interactive whiteboards.

Thus, Blender is not only a 3D modeling tool, but also a powerful educational resource for visualizing mathematical concepts and developing students' systematic and visual thinking. The animated demonstration of the sphere helps students understand the connection between the formula and the real geometric model, which significantly increases the effectiveness of learning the material.

The theoretical analysis confirmed that effective visualization of the concepts of area and diameter in school geometry is possible only if digital environments are used that allow for real dynamic modeling of geometric relationships. Among all existing tools, the greatest potential is demonstrated by the GeoGebra, Desmos environments and engineering modeling software platforms (e.g., Blender, Tinkercad, MATLAB). It is these resources that create the conditions for the transition from traditional two-dimensional illustration to interactive three-dimensional digital modeling, which is the basis of the concept of a digital twin. Results analytical comparison presented in Table 1.

Table 1
Comparison digital environments suitable for modeling concepts areas and diameter

Digital environment	Modeling type	Ability to change parameters in real time	3D visualization support	Suitability for creating a digital twin	Level of pedagogical integration
GeoGebra	Dynamic geometry (2D/3D)	High	Yes	Limited but possible	High
Desmos	Analytical graphical modeling	High	No (2D)	Partial	High
Blender	3D modeling of objects	High	Yes	High	Average
Tinkercad	3D object designer	Medium	Yes	High	Average
MATLAB	System modeling	High	Yes	High	Low

Source: complied by authors

The visualization of the comparison data can be presented in the form of a graph (Figure 3), which shows the relationship between the level of interactivity and the potential for creating a digital twin. The assessment of the level of interactivity and the potential for creating a digital twin was carried out based on the analysis of scientific publications Lexman and Baral [27], Carlin et al. [28], Uhlenkamp et al. [29].

The results of the analysis show that the most optimal for educational purposes is to use GeoGebra as a basic environment for demonstrating mathematical relationships between area and diameter, in combination with engineering programs such as Tinkercad or Blender, which allow to create a visually and technologically realistic digital twin. This approach provides a gradual transition from theoretical visualization to practical digital modeling, which strengthens the interdisciplinary nature of learning (combining geometry, computer science and technology).

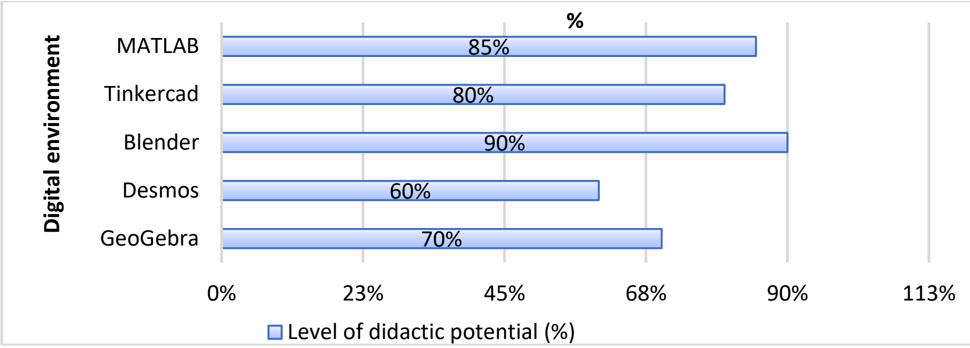

Fig. 3. Assessment of the didactic potential of digital environments in terms of the level of interactivity and suitability for creating a digital twin
Source: complied by authors according to [27–29]

Thus, it is the synthesis of mathematical and technological approaches that creates conditions for the development of students' spatial thinking, increasing the level of cognitive activity, and mastering abstract concepts through dynamic models.

As a result of the analytical study, it can be argued that digital resources in geometry teaching act as a catalyst for the transition from reproductive to productive learning. They open up new opportunities for the development of spatial imagination, critical thinking, and also contribute to the formation of students' skills of independent discovery of knowledge. Based on the analysis, the feasibility of creating a digital twin of a geometric object as the next stage in the development of visualization technologies in mathematics teaching was substantiated.

The interactive visualization developed in the work, presented in the form of an HTML application (Figure 4), is an example of a digital twin of a geometric object – a circle, which allows exploring the relationship between radius, diameter and area in real time. This resource creates a virtual copy of a mathematical object that responds to changes in parameters by the user and thereby reflects the pattern described by the formula $S = \pi r^{2.}$ Thanks to this approach, the abstract dependence between quantities acquires a specific visual content, which makes the process of cognition more visual and exploratory.

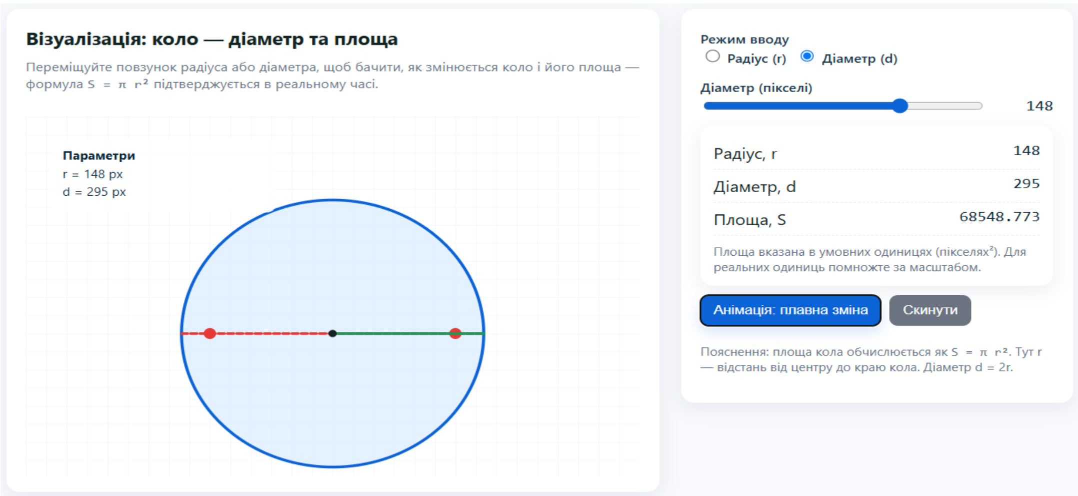

Fig. 4. Interface of the digital twin of a circle (HTML application): central area – dynamic image of a circle with radius and diameter; right panel – slider for changing the radius, switch for “radius/diameter” mode and output of numerical values; “Animation” button starts automatic change of parameters
Source: complied by authors

The program interface is implemented in the format of an interactive window divided into two parts: a visual field with a circle and a parameter control panel. The central part of the screen displays a circle, radius, diameter, and center, which change dynamically when the slider is moved. The student can choose the operating mode – by radius or by diameter – and instantly observe how the shape of the object, its dimensions, and numerical indicators change. The right part of the interface displays the values of the parameters: radius, diameter, and area calculated by the formula. There is also an "Animation" button, which allows to automatically vary the parameters and see the continuous process of changing the circle. Thus, the interface is built so that the user can not only observe, but also actively interact with the model, changing its characteristics according to their own choice.

In a pedagogical context, such a program serves as a digital experiment, within which the student can verify mathematical laws on his own. Unlike static illustrations in textbooks, a digital twin provides direct interaction with a geometric object – each change in a parameter is accompanied by an instant reaction of the system, which stimulates the development of cause-and-effect thinking. As a result, the student not only "knows" the formula for the area of a circle, but also understands why it has this form, because he sees how a change in the radius proportionally affects the area.

From a didactic point of view, the program has great potential for use at different stages of the educational process. At the stage of explaining new material, it helps to form an initial idea of the relationship between radius and area. When consolidating knowledge, it can serve as a simulator for independent research or laboratory observations. In addition, the interactive nature of the environment helps to increase motivation for learning, because students perceive mathematical concepts not as abstract formulas, but as living processes that can be controlled and analyzed.

The created HTML visualization is not just a technical tool, but a pedagogical digital twin that integrates a mathematical model, graphic dynamics, and analytical thinking. It provides a deep understanding of the concepts of area and diameter, develops spatial imagination, and supports interdisciplinary connections between geometry, computer science, and technology. This approach demonstrates that digital twins can be effectively used in school education to form an investigative thinking style and develop students' cognitive activity.

The use of a digital twin of a circle in the process of studying geometry opens up new opportunities for organizing students' cognitive activity. The methodology of its application is based on the principles of interactivity, clarity and research orientation of learning, which ensures the active participation of students in the process of discovering mathematical patterns. The goal is not only to master the formula for the area of a circle, but also to gain a deep understanding of its content through direct observation of changes in parameters in the digital environment.

It is advisable to build the educational process according to a stage structure (Figure 5). At the motivational stage, the teacher offers students a simple question: “How will the area of a circle change if its diameter is doubled?” Instead of a theoretical explanation, a digital twin is immediately demonstrated – an interactive model of a circle, in which one can change the size of the figure in real time using a slider. The visual response of the system arouses interest and creates a situation of cognitive surprise: students see that the area does not increase by half, but by four times. In this way, internal motivation for research is formed.

At the stage of learning new material, students interact with the digital twin independently. They change the radius or diameter of the circle, observe the corresponding changes in the area, and record the results in the observation table. After several experiments, students independently conclude that the area depends on the radius in quadratic fashion. This process transforms a simple demonstration into a research activity in which the student is not a passive observer, but an active discoverer of the pattern.

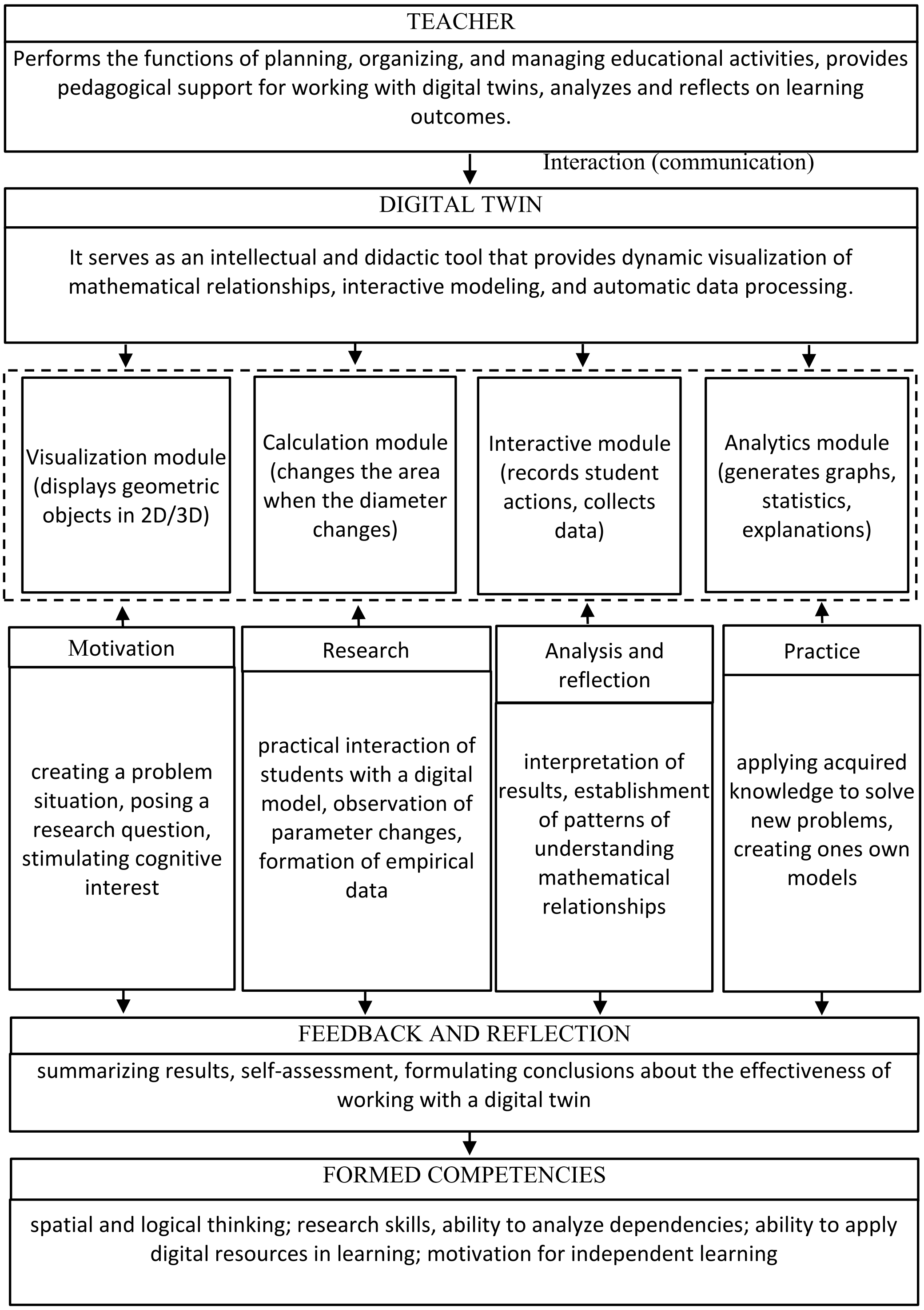

Fig. 5. Structural and functional diagram of the phased use of a digital twin in a geometry lesson: motivational, research, analytical and practical stages
Source: complied by authors

The stage of comprehension and consolidation involves analytical work with the results. The teacher organizes a discussion: why does changing the radius affect the area in a square? How can this be explained geometrically? At this stage, it is important to move from visual perception to theoretical generalization. Students can compare graphical and numerical data, build dependencies in the form of functions in Desmos or GeoGebra, which provides cross-curricular integration of geometry and algebra.

The practical stage includes individual or group tasks. Students can create their own modification of the digital twin, for example, model a circle that turns into a cylinder or a sphere when changing parameters. Such activities develop not only mathematical, but also technical and creative competencies, which is consistent with the concept of STEM education.

Methodologically, it is important to maintain a balance between the technological and didactic components. A digital tool should not replace thinking, but only enhance it. The teacher should guide students' activities through a system of questions, comparisons, and generalizations. For example:

– what happens to the diameter when the radius increases by 50%?

– how will the area change if the radius is halved?

– is it possible to plot a graph of area versus diameter?

Such tasks form causal, analytical, and functional thinking, and develop mathematical modeling and argumentation skills.

In addition to educational purposes, the digital twin also has educational potential. Working with interactive models forms a culture of research, accuracy of observations, and independence in testing hypotheses. Students learn not to take formulas on faith, but to test them through an experiment, which corresponds to the idea of a constructivist approach in education.

Thus, the technique of using a digital twin of a circle when studying the concepts of area and diameter is based on a combination of visual exploration, analytical thinking, and independent experimentation. It provides a gradual transition from empirical observation to formalized mathematical generalization, increases motivation for learning, and contributes to the formation of a deep, conscious understanding of geometric patterns.

5. Discussion

In the process of discussing the results of the study, a number of theoretical and practical questions arise that determine the prospects for the further development of digital technologies in geometry teaching. The analysis shows that digital environments do create unique conditions for the formation of spatial thinking, but their pedagogical effectiveness directly depends on the level of didactic integration. This conclusion is consistent with the position of Putra et al. [30], who emphasize that technology cannot replace the teacher, but can radically strengthen his methodological tools.

On the one hand, the results indicate the obvious advantage of digital tools in ensuring visuality, interactivity and experimentality of learning. Students have the opportunity not only to see the mathematical dependence, but also to “feel” it through changing parameters, which contributes to the formation of an intuitive understanding of geometric laws. On the other hand, excessive technologization of the process can lead to formal assimilation without deep understanding, if the teacher does not ensure the transition from the visual to the analytical level of thinking.

The issue of the balance between digital and traditional didactics remains debatable. Some researchers [31, 32] argue that the digital environment creates a “new paradigm of mathematics education” in which the student operates not within the confines of a textbook, but in a virtual field of inquiry. Dancsa et al [33] and Mucundanyi and Woodley [34] believe that digital resources should remain an auxiliary tool that only supports the logic of traditional learning. In practice, a synthetic approach seems appropriate, when the digital twin does not replace the content, but reveals it through visual exploration.

In industrial and technical systems, the term "digital twin" means an exact virtual copy of a physical object that reproduces its behavior. In pedagogy, the digital twin acquires a cognitive - didactic meaning: it not only reflects the object, but also creates conditions for its cognition. In this context, the digital twin of a circle is not just a visual model, but an interactive system that responds to the actions of the student, provides cause-and-effect relationships and forms exploratory thinking.

The issue of evaluating the effectiveness of digital environments remains controversial. The study identified the relative advantage of GeoGebra for educational purposes. However, the quantitative assessment of the impact of a digital twin on the level of mastery of concepts requires empirical confirmation, for example, through a pedagogical experiment with control and experimental groups. This is especially relevant in the context of the implementation of STEM education and the competency approach.

The use of digital twins causes the teacher to move from the position of a "source of knowledge" to a facilitator of the educational process, who organizes research activities. The student, in turn, becomes a co-author of the educational experience, rather than a passive observer. This approach changes the very structure of pedagogical interaction, bringing it closer to the concepts of constructivism and research-based learning.

From a practical point of view, the implementation of digital twins in geometry teaching requires taking into account several factors. First, the availability of software: most platforms (GeoGebra, Desmos, LearningApps) are free or have free versions, which makes them suitable for mass use. At the same time, more complex environments, such as MATLAB or Blender, require licenses and more powerful hardware, which may limit their use in schools with outdated computer technology. Second, the level of technical skills of teachers is critical: for the effective integration of digital twins, the teacher must have not only basic digital literacy, but also the ability to work with interactive models, configure parameters and interpret results. This necessitates the need for systematic professional development and methodological support. Third, it is important to take into account the time constraints of the lesson: working with a digital twin requires careful planning so that research activities do not turn into technical demonstrations. Therefore, it is advisable to develop short, focused tasks that easily fit into the structure of the lesson.

Currently, not all schools have the technical ability to provide the use of complex 3D environments such as Blender or MATLAB. Therefore, it is important to consider the principle of pedagogical feasibility and technological simplicity: effective learning can occur even with the help of simple but well-integrated tools such as GeoGebra or Desmos.

Thus, digital twins have a powerful potential in teaching geometry, but their effectiveness is determined by the implementation methodology, role interaction of participants, and pedagogical purpose of use. A promising direction for further research is the development of integrated learning models, where the digital twin will act not only as a visualization tool, but also as a means of forming mathematical thinking through research activities.

6. Conclusion

The conducted research allowed to theoretically and methodologically substantiate the feasibility of using digital resources in the process of teaching geometry, in particular when studying the concepts of area and diameter. The analysis of scientific sources and modern digital environments showed that digital technologies not only diversify the forms of presentation of educational material, but also contribute to a deep cognitive restructuring of students' mathematical thinking, making it visual-algorithmic and research-based.

Digital visualization is an effective means of integrating practical experience with theoretical knowledge. It provides the opportunity to directly observe mathematical patterns in dynamics, which contributes to the formation of cause-and-effect relationships and improves the quality of understanding of concepts.

The use of dynamic geometry environments creates conditions for the transition from static to interactive presentation of educational material. This enhances students' cognitive activity and stimulates the development of spatial imagination, which is an important component of mathematical competence.

The concept of a digital twin of a geometric object has proven its didactic effectiveness as a tool that combines a mathematical model, graphic dynamics, and the possibility of experimental student interaction with educational content. The created digital environment of a digital twin of a circle provides a deeper understanding of the relationship between radius, diameter, and area, transforming an abstract formula into a visually perceptible pattern.

The methodology of using digital twins in teaching geometry should be based on the principles of interactivity, research orientation, and pedagogical feasibility. Its effectiveness depends on the role of the teacher as a facilitator, who organizes students' activities through experimentation, reflection, and analysis of results.

The developed structural-functional model demonstrates the possibility of harmonizing technological, cognitive and didactic components of the educational process. The digital environment performs not only the role of a technical tool, but also the function of a cognitive environment in which the student moves from empirical observation to formalized theoretical generalization. From a pedagogical point of view, the digital twin is a catalyst for the transition from a reproductive to a productive type of learning, as it stimulates independence, research activity and critical thinking.

At the same time, it should be emphasized that the conclusions of this study are of a theoretical and methodological nature. The proposed method of using the digital twin of the circle has not been experimentally tested in the conditions of the real educational process; empirical validation of its effectiveness remains a task for further research.

Prospects for further research include developing methods for assessing the impact of digital twins on the level of student achievement, as well as expanding the model of their use for other mathematical concepts and subject areas in the context of STEM education.

Thus, the results of the study confirm that the introduction of digital twins in geometry teaching is an effective direction of development of modern education, which combines technological capabilities with pedagogical principles of meaningful learning. Such an approach contributes to the formation of a new type of mathematical thinking – activity-visual, which meets the challenges of the digital age.

Author Contributions

Conceptualization, L.H. and L.K.; methodology, L.H.; validation, L.H. and L.K.; formal analysis, L.H.; investigation, L.H. and L.K.; resources, L.K.; data curation, L.H.; writing—original draft preparation, L.H.; writing—review and editing, L.H. and L.K.; visualization, L.H.; supervision, L.K.; project administration, L H. All authors have read and agreed to the published version of the manuscript.

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Data Availability Statement

No new datasets were created or analyzed in this study. The research is based on publicly available scientific literature, open educational platforms, and methodological documents that are cited within the article.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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